



## IMPORTANCE OF MATHEMATICAL MODELING IN TEACHING MATHEMATICS

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### ABSTRACT

Today, in engineering departments, mathematics courses such as calculus, linear algebra and differential equations are taught by mathematicians. Therefore, in their classroom teaching there are few or no applications of the concepts to real world problems. Most of the times, students do not know whether the concepts or rules taught in these courses will be used extensively in their majors or not. This situation holds true for all engineering disciplines. The real-life application of mathematics will be appreciated by students when mathematical modeling of real world problems are tackled. In this paper some mathematical concepts are chosen and their applications to real-life problems are emphasized along with mathematical modeling for applications of mathematics.

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### INTRODUCTION

The language of universe is mathematics. Without it to understand how the nature behaves is almost impossible. Especially in engineering education understanding mathematics thoroughly is a must. Today, in engineering departments, mathematics courses such as calculus, linear algebra and differential equations are taught by mathematicians. Therefore, in their classroom teaching, there are few or no applications of the concepts to real-world problems. Most of the times, students do not know whether the concepts or rules taught in these courses will be used extensively in their majors or not. The real-life application of mathematics will be appreciated by students when mathematical modeling of real world problems are tackled. The author of this paper believes that unless math is applied to a real life problems it will not be completely appreciated by student. Engineering or science students would like to see a solid application of the concepts to physical world, rather than having an abstract concept.

The author also claims that the main application of mathematics comes with mathematical modeling of real-life problems. In the following part of the paper, some the most important concepts will be considered from the point of physical applications. The list is not necessarily complete, but will be a seed for a betterment of teaching mathematics courses.

#### Limit: Ultimate Reality, Goal, Target

The most important concepts in calculus is the limit. Without understanding limit properly, one will have difficulty in derivatives and integration which are the pillar of calculus. The following expression does not mean too much for a student, unless an example of a relevant physical phenomenon is given to demonstrate its application.

$$\lim_{x \rightarrow a^+, a^-} f(x)$$

If we take the peak of a mountain as our ultimate point (goal) then reaching the peak of mountain from left or right (i.e. from  $a^-$  or from  $a^+$ ) will give students a good practical understanding of limit (Figure 1). We may be extremely close to the peak, but not necessarily just right at the peak itself (Thomas, 2010).

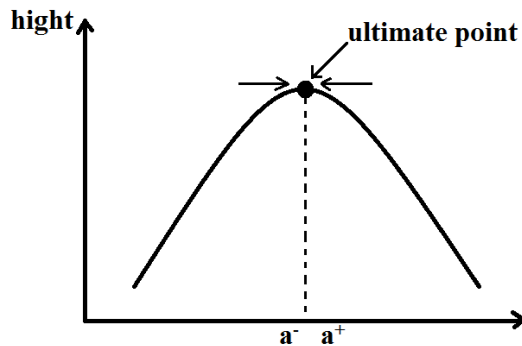


Fig. 1. Physical meaning of limit

**Derivative: Rate of change, how fast something is changing**

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

The change of a dependent variable such as temperature, pressure, distance, velocity, and so on with respect to a tiny changes in an independent variable, for example time, is very common in the physical world. Average change is shown in Figure 2. In engineering, one generally is interested in instantaneous change i.e. the slope of the tangent at that point. For an automobile, the average speed between a time span of  $t_2$  and  $t_1$ , as well as instantaneous speeds at  $t_1$  and  $t_2$  times are given in Figure 3 with necessary equations. This kind of approach to the concept of derivative would be much more beneficial for students (Bird, 2002).

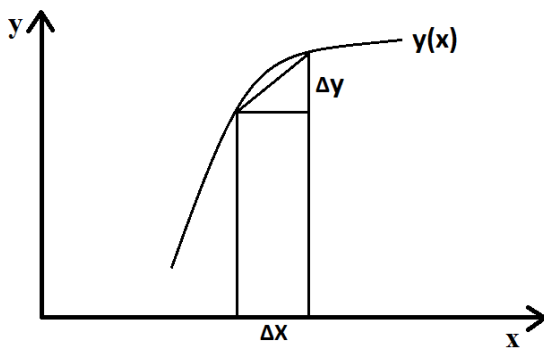


Fig. 2. Average change:  $\Delta y/\Delta x$

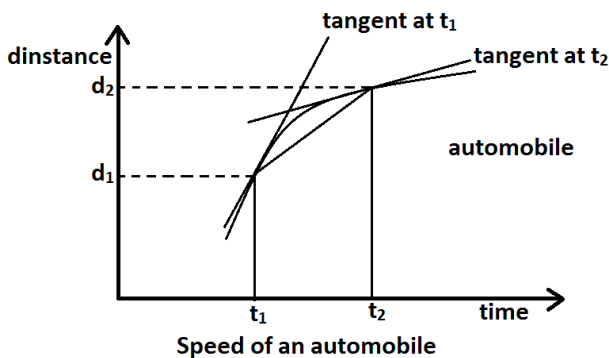


Fig. 3. Instantaneous speeds at  $t_1$  and  $t_2$

Average speed =  $(d_2-d_1)/(t_2-t_1)$ , instantaneous speed at  $t_1 = [d(d)/dt]_{t_1}$  and instantaneous speed at  $t_2 = [d(d)/dt]_{t_2}$

**Integration: Growth, Summing, Growth of a Snowball**

The growth of a snowball, the growth of population are good examples of integration. As can be seen in Figure 4, integration is simply the summation of small parts. An example of mass and energy balances over a small slice of a integral reactor and then the extension of this balance for the entire reactor is a significant process for the understanding of the concept of integration. Therefore, integration is step by step addition (Gültekin, 1997; Gültekin, 2012; Gültekin, 2013; Thomas, 2010 and Bird, 2002).

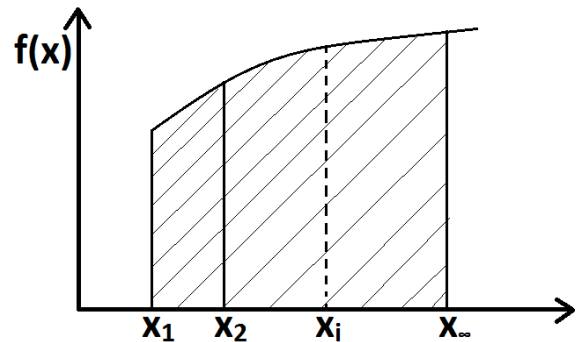


Fig. 4. Understanding of integration as an area under the curve

$$\int_{x_1}^{x_3} f(x) dx = \sum_{i=1}^{\infty} f(x_i) \Delta x_i = \text{Area under the curve}$$

Differential equations: Pieces of a puzzle

$$\frac{dy}{dx} = f(x)$$

This expression is mathematically correct, but still students have a hard time understanding it. Differential equations cannot be used as such in engineering applications! For engineering and science students, a radioactive disintegration can be given as an example of differential equations. This mathematical equation also signifies the concept of derivative (change of mole number of radioactive component over an infinitesimal time). A situation for radioactive disintegration is summarized in Figure 5 (Fogler, 2006).

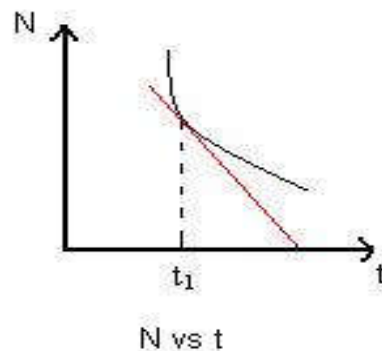


Fig. 5. N vs time

Disintegration rate

$$-\frac{dN}{dt} = kN$$

**k=Disintegration constant**

We now have to find how N changes with time

$$-\frac{dN}{dt} = k dt \Rightarrow \ln N = -kt + \ln N_o \Rightarrow \frac{N}{N_o} = e^{-kt} \Rightarrow N = N_o e^{-kt}$$

**Differential equation → Integrated forms**

**Taylor series: Linearization: Approximate calculations**

Engineering processes in mathematical modeling result in *non-linear* equations (either algebraic or differential from). In order to solve, for example a non-linear differential equation, for approximate solution the equation is to be linearized through Taylor Series expansion. These kinds of linearization are extremely important in process dynamics and control. The physical understanding of Taylor expansion is given in Figure 6.

The first two terms in the Taylor Expansion are good enough for the approximate solution (Gültekin, 2012; Stephanopoulos, 1984 and Dale, 2011).

$$f(x) = f(x_o) + f'(x_o) \left. \frac{(x-x_o)}{1!} + f''(x_o) \frac{(x-x_o)^2}{2!} \right|_{x_o} + \dots + \frac{f^{(n)}(x-x_o)^n}{n!}$$

In most of the time we have to linearize Arrhenius Expression  $k = k_o e^{-E/RT}$  (non-linear in T)

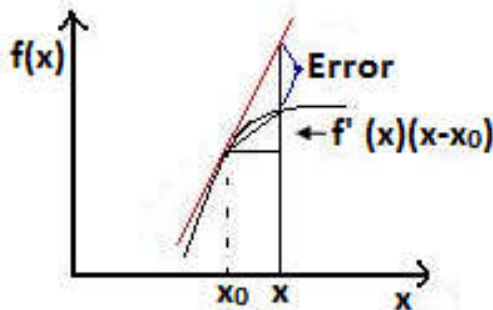


Fig. 6. Physical understanding of Taylor Expansion

$$y_{mean} = \frac{\int_{x_1}^{x_3} y(x) dx}{x_3 - x_1}$$

**Mean Value Theorem**

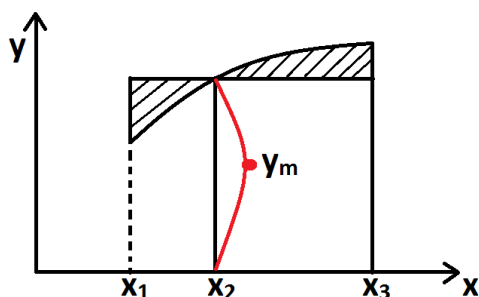


Fig. 7. Graphical representation of Mean Value Theorem

In chemical engineering, for example, certain values (e.g.  $C_p$ =specific heat,  $v$ =velocity, and so on) change with changes in some other variables. Specific heat,  $C_p=f(T)$  or velocity in a pipe,  $v = f(A)$ . The geometrical understanding of mean value theorem is given in Figure 7. In Figures 8 and 9, we can see how to calculate mean specific heat  $C_{pm}$  for the temperature range of  $T_1$  and  $T_2$  (Bird, 2002) and mean velocity (McCabe, 1993), respectively.

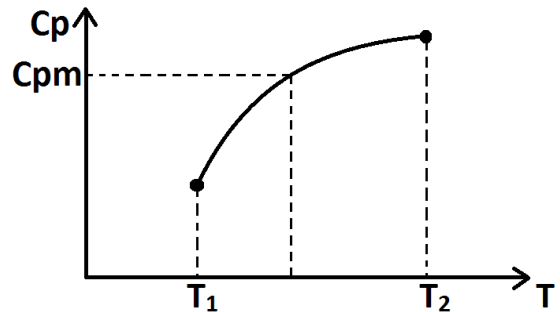


Fig. 8. Change of specific heat with temperature

$$C_{pm} = \frac{\int_{T_1}^{T_2} C_p(T) dt}{T_2 - T_1}$$

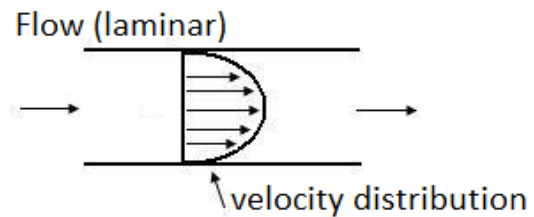


Fig. 9. Velocity distributions in a pipe for laminar flow

$$v_{mean} = \frac{\int_A v dA}{A}$$

**Mathematical Modeling**

The math modeling is simply representation of the real-life phenomenon by mathematical equations. This means that one can predict the result of a physical or chemical process without going to laboratory. Just understanding of the phenomenon and mathematics is good enough for mathematical modeling. No doubt that applications of mathematics is a good practice to use in mathematical modeling. It is highly recommended that for a good appreciation of mathematics, mathematical modeling should be a core course in all high school all over the world. So that once students are at the university, she/he will appreciate the importance of mathematics for the major chosen.

**Conclusions**

Mathematics courses given in engineering departments must not only focus on concepts, but must also apply these concepts to real life engineering problems, so that the students understand the importance of mathematics in engineering. Mathematical modeling is a good area for the application of

mathematics to real-life problems. Mathematical modeling could be a must course in high schools before student comes to university.

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