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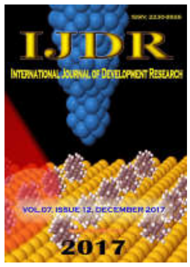

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SOME PROPERTIES OF FUZZY QUASI IDEALS IN TERNARY SEMIGROUPS

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ABSTRACT

D.H. Lehmer introduced the theory of ternary algebraic system in 1932 and iIn 1965, F.M. Sioson studied ideal theory in ternary semigroups. In this paper we proved some properties of fuzzy quasi ideals in ternary semigroups. It is prove that "A non-empty subset A of a ternary semigroup T is a quasi ideal in T if and only if the characteristic function C_A of A is a fuzzy quasi ideal in T".

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INTRODUCTION

In 1965, F.M. Sioson studied ideal theory in ternary semgroups. He also introduced the notion of regular ternary semigroups and characterized them by using the notion of quasi ideals. In 1995, V.N. Dixit and S. Dewan[4] introduced and studied the properties of ideals in ternary semigroups. The concept of ideals is an interesting and important idea in many algebraic structures. Several researches have characterized many type of ideals on the algebraic structures such as: Iampan studied the concept and gave some characterizations of (0-)minimal and maximal ordered bi-ideals in ordered G-semigroups and he developed the concept of ideal extensions in ternary semigroups. The connection between an ideal extensions and semilattice congruences in ternary semigroups is considered. Nezhad gave several characterizations of strongly prime ideals of commutative integral domains. Shabir, Jun and Bano introduced and studied the prime, strongly prime, semiprime and irreducible fuzzy bi-ideals of semigroups. They characterized those semigroups for which each fuzzy bi-ideal is semiprime and also characterized those semigroups for which each fuzzy bi-ideal is strongly prime.

1. Preliminaries and Basic Results

1.1 Definition: A non-empty set T is said to be ternary semigroup if there exists a ternary operation $\cdot : T\times T\times T \rightarrow T$ written as $(a,b,c) \rightarrow a.b.c$ satisfies the following identity $(abc)de = a(bcd)e = ab(cde)$ for any $a,b,c,d,e \in T$.

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Example 1: Let $T = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ then T is a ternary semigroup under usual multiplication.

1.2 Definition: A non-empty subset A of a ternary semigroup T is called a ternary sub semigroup of T if $AAA \subseteq A$.

1.3 Definition: A non-empty subset A of a ternary semigroup T is called a left (right, lateral) ideal in T if $TTA \subseteq A$($ATT \subseteq A$, $TAT \subseteq A$).

1.4 Definition: A non-empty subset A of a ternary semigroup T is called a two sided ideal of T if it is both left and right ideal in T.

1.5 Definition: A non-empty subset A of a ternary semigroup T is called a ideal in T if it is left, right and lateral ideal in T.

1.6 Definition: A non-empty subset T of a ternary semigroup T is called a quasi ideal in T if $(ATT) \cap (TAT) \cap (TTA) \subset A$ and $(ATT) \cap (TTATT) \cap (TTA) \subset A$.

1.7 Definition: A ternary sub semigroup A of a ternary semigroup T is said to be a bi-ideal in T if $ATATA \subset A$.

1.8 Definition: Let T be a non-empty set. A fuzzy subset of a ternary semigroup T is a function $\mu : T \to [0, 1]$.

1.9 Definition: Let μ be a fuzzy subset of a non-empty set T for any $t \in [0, 1]$, the subset $\mu_t = \{x \in T : \mu(x) \geq t\}$ of T is called a level set of μ.

1.10 Definition: For any two fuzzy subsets μ_1 and μ_2 of a non-empty set T, the union and the intersection of μ_1 and μ_2 denoted by $\mu_1 \cup \mu_2$ and $\mu_1 \cap \mu_2$ are fuzzy subsets of T and defined as $(\mu_1 \cup \mu_2)(x) = \max\{(\mu_1(x), \mu_2(x)\}$ $= \mu_1(x) \vee \mu_2(x)$ and $(\mu_1 \cap \mu_2)(x) = \min\{(\mu_1(x), \mu_2(x)\} = \mu_1(x) \wedge \mu_2(x)$ for all $x \in T$ Where $\vee$ denotes maximum or supremum and $\wedge$ denotes minimum or infimum.

1.11 Definition: Let μ_1, μ_2 and μ_3 are any three fuzzy sets of a ternary semigroup T. Then their fuzzy product

$\mu_1 \circ \mu_2 \circ \mu_3$ is defined by

$$(\mu_1 \circ \mu_2 \circ \mu_3)(a) = \begin{cases} \bigvee_{a=xyz} \{\mu_1(x) \wedge \mu_2(y) \wedge \mu_3(z)\} & \text{if } a \text{ is expressible as } a = xyz \text{ for all } x,y,z \in T \\ 0 & \text{otherwise} \end{cases}$$

1.12 Definition: A fuzzy set μ of a ternary semigroup T is called a fuzzy ternary subsemigroup of T if $\mu(xyz) \geq \{\mu(x) \wedge \mu(y) \wedge \mu(z)\}$ for all $x, y, z \in T$.

1.13 Definition: A fuzzy ternary sub semigroup μ of a ternary semigroup T is called a fuzzy bi-ideal in T if $\mu(xmynz) \geq \{\mu(x) \wedge \mu(y) \wedge \mu(z)\}$ for all $x, y, z, m, n \in T$

1.14 Definition: A fuzzy set μ of a ternary semigroup T is called a fuzzy left (right, lateral) ideal in T if $\mu(xyz) \geq \mu(z)$, ($\mu(xyz) \geq \mu(x)$, $\mu(xyz) \geq \mu(y)$) for all $x, y, z \in T$

1.15 Definition: A fuzzy set μ of a ternary semigroup T is a fuzzy ideal in T if it is fuzzy left , right and lateral ideal in T.

1.16 Definition: Let A be a non-subset of a ternary semigroup T. Then the characteristic function of A is defined by

$$C_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

We denote the characteristic function C_T of T. i.e., $T = C_T$ thus $T(x) = 1$ for all $x \in T$.

1.17 Definition: A fuzzy set μ of a ternary semigroup T is called a fuzzy quasi ideal of T if $(T \circ T \circ \mu) \cap (T \circ \mu \circ T) \cap (\mu \circ T \circ T) \subseteq \mu$ and $(T \circ T \circ \mu) \cap (T \circ T \circ \mu \circ T \circ T) \cap (\mu \circ T \circ T) \subseteq \mu$.

i.e., $[(T \circ T \circ \mu) \cap (T \circ \mu \circ T) \cap (\mu \circ T \circ T)](a) \leq \mu(a)$ and

$[(T \circ T \circ \mu) \cap (T \circ T \circ \mu \circ T \circ T) \cap (\mu \circ T \circ T)](a) \leq \mu(a)$

1.18 Definition: A subset A of a ternary semigroup T is said to be a prime ideal in T if $xyz \in A$ implies $x \in A$ or $y \in A$ or $z \in A$.

1.19 Definition: A fuzzy ideal μ of a ternary semigroup T is called a fuzzy weakly completely prime ideal in T if $\mu(x) \geq \mu(xyz)$ or $\mu(y) \geq \mu(xyz)$ or $\mu(z) \geq \mu(xyz)$ for all $x, y, z \in T$.

1.20 Definition: A fuzzy ideal μ of a ternary semigroup T is called a fuzzy prime ideal in T if $\inf \mu(xyz) \geq \max\{\mu(x), \mu(y), \mu(z)\}$ for all $x, y, z \in T$.

1.21 Proposition [3]: If μ_1 μ_2 μ_3 μ_4 and μ_5 are fuzzy subsets of a non-empty set T then

(i) $\mu_1 \cap (\mu_2 \cup \mu_3) \cap \mu_4 = (\mu_1 \cap \mu_2 \cap \mu_4) \cup (\mu_1 \cap \mu_3 \cap \mu_4)$.
(ii) $(\mu_1 \cup \mu_2) \circ \mu_3 \circ \mu_4 = (\mu_1 \circ \mu_3 \circ \mu_4) \cup (\mu_2 \circ \mu_3 \circ \mu_4)$.
(iii) $\mu_1 \circ (\mu_2 \cup \mu_3) \circ \mu_4 = (\mu_1 \circ \mu_2 \circ \mu_4) \cup (\mu_1 \circ \mu_3 \circ \mu_4)$.
(iv) $\mu_1 \circ \mu_2 \circ (\mu_3 \cup \mu_4) = (\mu_1 \circ \mu_2 \circ \mu_3) \cup (\mu_1 \circ \mu_2 \circ \mu_4)$.
(v) $(\mu_1 \circ \mu_2 \circ \mu_3) \circ \mu_4 \circ \mu_5 = \mu_1 \circ (\mu_2 \circ \mu_3 \circ \mu_4) \circ \mu_5 = \mu_1 \circ \mu_2 \circ (\mu_3 \circ \mu_4 \circ \mu_5)$.

1.22 Proposition: If μ_1 and μ_2 are two fuzzy subsets of a non-empty set T then
(i) $((\mu_1 \cap \mu_2) \circ T \circ T) \subseteq (\mu_1 \circ T \circ T) \cap (\mu_2 \circ T \circ T)$
(ii) $(T \circ (\mu_1 \cap \mu_2) \circ T) \subseteq (T \circ \mu_1 \circ T) \cap (T \circ \mu_2 \circ T)$
(iii) $(T \circ T \circ (\mu_1 \cap \mu_2)) \subseteq (T \circ T \circ \mu_1) \cap (T \circ T \circ \mu_2)$.

Proof: (i) Let $x \in T$. If $x \neq pqr$ for any $p, q, r \in T$ then $((\mu_1 \circ T \circ T) \cap (\mu_2 \circ T \circ T))(x) = 0 =$ $((\mu_1 \cap \mu_2) \circ T \circ T)(x)$.

If $x = pqr$ for any $p, q, r \in T$ then

$$\begin{aligned}
((\mu_1 \cap \mu_2) \circ T \circ T)(x) &= \vee_{x=pqr} \{(\mu_1 \cap \mu_2)(p) \wedge T(q) \wedge T(r)\} \\
&= \vee_{x=pqr} \{\mu_1(p) \wedge \mu_2(p) \wedge 1 \wedge 1\} \\
&= \vee_{x=pqr} \{\mu_1(p) \wedge \mu_2(p)\} \\
&\leq \{\vee_{x=pqr} \{\mu_1(p)\}\} \wedge \{\vee_{x=pqr} \{\mu_2(p)\}\} \\
&= \{\vee_{x=pqr} \{\mu_1(p) \wedge 1 \wedge 1\}\} \wedge \{\vee_{x=pqr} \{\mu_2(p) \wedge 1 \wedge 1\}\} \\
&= \{\vee_{x=pqr} \{\mu_1(p) \wedge T(q) \wedge T(r)\}\} \wedge \{\vee_{x=pqr} \{\mu_2(p) \wedge T(q) \wedge T(r)\}\} \\
&= (\mu_1 \circ T \circ T)(x) \wedge (\mu_2 \circ T \circ T)(x) \\
&= ((\mu_1 \circ T \circ T) \cap (\mu_2 \circ T \circ T))(x)
\end{aligned}$$

$$((\mu_1 \cap \mu_2) \circ T \circ T)(x) \leq ((\mu_1 \circ T \circ T) \cap (\mu_2 \circ T \circ T))(x)$$

Therefore $((\mu_1 \cap \mu_2) \circ T \circ T) \subseteq ((\mu_1 \circ T \circ T)\ T$.

Similarly we can prove that

(ii) $(T \circ (\mu_1 \cap \mu_2) \circ T) \subseteq (T \circ \mu_1 \circ T) \cap (T \circ \mu_2 \circ T)$
and (iii) $(T \circ T \circ (\mu_1 \cap \mu_2)) \subseteq (T \circ T \circ \mu_1) \cap (T \circ T \circ \mu_2)$.

2 Main Results

2.1 Theorem: If μ is a fuzzy left ideal in a ternary semigroup T if and only if $T \circ T \circ \mu \subseteq \mu$.

Proof: Let μ be a fuzzy left ideal in a ternary semigroup T. Then we have $\mu(xyz) \geq \mu(z)$.

Consider $(T \circ T \circ \mu)(a) = \bigvee_{a=xyz} \{T(x) \wedge T(y) \wedge \mu(z)\}$

$$= \bigvee_{a=xyz} \{1 \wedge 1 \wedge \mu(z)\}$$

$$= \bigvee_{a=xyz} \mu(z)$$

$$\leq \bigvee_{a=xyz} \mu(xyz) \quad (\because \mu \text{ is a fuzzy left ideal of } T)$$

$$= \mu(a).$$

$(T \circ T \circ \mu)(a) \leq \mu(a)$

$(T \circ T \circ \mu) \subseteq \mu$ for all $a \in T$.

If $a \neq xyz$ then $(T \circ T \circ \mu)(a) = 0 \leq \mu(a) \Rightarrow (T \circ T \circ \mu)(a) \leq \mu(a)$

Therefore $T \circ T \circ \mu \subseteq \mu$.

Conversely, assume that $T \circ T \circ \mu \subseteq \mu$. Then for any $x, y, z \in T$,

we have $\mu(xyz) \geq (T \circ T \circ \mu)(xyz)$

$$= \bigvee_{xyz=abc} \{T(a) \wedge T(b) \wedge \mu(c)\}$$

$$= \{T(x) \wedge T(y) \wedge T(z)\}$$

$$= 1 \wedge 1 \wedge \mu(z)$$

$$= \mu(z)$$

$\mu(xyz) \geq \mu(z)$

Therefore μ is a fuzzy left ideal in T.

2.2 Theorem: A fuzzy set μ of a ternary semigroup T is a fuzzy right (lateral) ideal in T if and only if $\mu \circ T \circ T \subseteq \mu$ $(T \circ \mu \circ T \subseteq \mu)$.

Proof: Similar to the proof of Theorem 1.22.

2.3 Result[9]: For any non-empty subsets A, B and C of a ternary semigroup T, we have (i) $C_A \circ C_B \circ C_C = C_{ABC}$ (ii) $C_A \cap C_B \cap C_C = C_{A \cap B \cap C}$.

2.4 Theorem: A non-empty subset A of a ternary semigroup T is a quasi ideal in T if and only if the characteristic function C_A of A is a fuzzy quasi ideal in T.

Proof: Assume that A is a quasi ideal in a ternary semigroup T.

Then we have $(ATT) \cap (TAT) \cap (TTA) \subseteq A$ and $(ATT) \cap (TTATT) \cap (TTA) \subseteq A$.

Let C_A be the characteristic function of A in T.

Consider $(C_A \circ T \circ T) \cap (T \circ C_A \circ T) \cap (T \circ T \circ C_A)$

$$= (C_A \circ C_T \circ C_T) \cap (C_T \circ C_A \circ C_T) \cap (C_T \circ C_T \circ C_A)$$

$$= C_{ATT} \cap C_{TAT} \cap C_{TTA}$$

$$= C_{(ATT) \cap (TAT) \cap (TTA)}$$

$(C_A \circ T \circ T) \cap (T \circ C_A \circ T) \cap (T \circ T \circ C_A) \subseteq C_A$

and $(C_A \circ T \circ T) \cap (T \circ T \circ C_A \circ T \circ T) \cap (T \circ T \circ C_A)$

$$= (C_A \circ C_T \circ C_T) \cap (C_T \circ C_T \circ C_A \circ C_T \circ C_T) \cap (C_T \circ C_T \circ C_A)$$

$$= C_{ATT} \cap C_{TTATT} \cap C_{TTA}$$

$= C_{(ATT)\cap(TTATT)\cap(TTA)}$

$(C_A \circ T \circ T) \cap (T \circ T \circ C_A \circ T \circ T) \cap (T \circ T \circ C_A) \subseteq C_A$

Hence C_A is a fuzzy quasi ideal in T.

Conversely, assume that C_A is a fuzzy quasi ideal in T.

Then we have $(C_A \circ T \circ T) \cap (T \circ C_A \circ T) \cap (T \circ T \circ C_A) \subseteq C_A$

and $(C_A \circ T \circ T) \cap (T \circ T \circ C_A \circ T \circ T) \cap (T \circ T \circ C_A) \subseteq C_A$

Let $x \in (ATT) \cap (TAT) \cap (TTA)$. Then

$C_A(x) \geq [(C_A \circ T \circ T) \cap (T \circ C_A \circ T) \cap (T \circ T \circ C_A)](x)$

$\geq [(C_A \circ C_T \circ C_T) \cap (C_T \circ C_A \circ C_T) \cap (C_T \circ C_T \circ C_A)](x)$

$\geq C_{(ATT)\cap(TAT)\cap(TTA)}(x) \qquad (\because x \in (ATT) \cap (TAT) \cap (TTA))$

$C_A(x) \geq 1 \Rightarrow x \in A$

Therefore $(ATT) \cap (TAT) \cap (TTA) \subseteq A$

and let $x \in (ATT) \cap (TTATT) \cap (TTA)$. Then

$C_A(x) \geq [(C_A \circ T \circ T) \cap (T \circ T \circ C_A \circ T \circ T) \cap (T \circ T \circ C_A)](x)$

$\geq [(C_A \circ C_T \circ C_T) \cap (C_T \circ C_T \circ C_A \circ C_T \circ C_T) \cap (C_T \circ C_T \circ C_A)](x)$

$\geq C_{(ATT)\cap(TTATT)\cap(TTA)}(x) \qquad (\because (ATT) \cap (TTATT) \cap (TTA))$

$C_A(x) \geq 1 \Rightarrow x \in A$

Therefore $(ATT) \cap (TTATT) \cap (TTA) \subseteq A$

Hence A is a quasi ideal in T.

2.5 Theorem: Let μ_1, μ_2 and μ_3 be fuzzy right , lateral and left ideals in a ternary semigroup T respectively. Then $\mu_1 \circ \mu_2 \circ \mu_3 \subseteq \mu_1 \cap \mu_2 \cap \mu_3$.

Proof: Given that μ_1, μ_2 and μ_3 be fuzzy right , lateral and left ideals in a ternary semigroup T respectively. Let $a \in T$ such that $a = xyz$ for all $x, y, z \in T$.

Consider $(\mu_1 \circ \mu_2 \circ \mu_3)(a) = \bigvee_{a=xyz} \{\mu_1(x) \wedge \mu_2(y) \wedge \mu_3(z)\}$

$\leq \bigvee_{a=xyz} \{\mu_1(xyz) \wedge \mu_2(xyz) \wedge \mu_3(xyz)\}$

$\leq \mu_1(a) \wedge \mu_2(a) \wedge \mu_3(a)$

$(\mu_1 \circ \mu_2 \circ \mu_3)(a) \leq (\mu_1 \cap \mu_2 \cap \mu_3)(a)$

Therefore $\mu_1 \circ \mu_2 \circ \mu_3 \subseteq \mu_1 \cap \mu_2 \cap \mu_3$.

If $a \neq xyz$ then $(\mu_1 \circ \mu_2 \circ \mu_3)(a) = 0 \leq (\mu_1 \cap \mu_2 \cap \mu_3)(a)$

Hence $\mu_1 \circ \mu_2 \circ \mu_3 \subseteq \mu_1 \cap \mu_2 \cap \mu_3$.

2.6 Theorem: Let μ_1 and μ_2 be fuzzy right and left ideals in a ternary semigroup T respectively. Then $\mu_1 \circ T \circ \mu_2 \subseteq \mu_1 \cap \mu_2$.

Proof: Given that μ_1 be fuzzy right ideal and μ_2 fuzzy left ideal in a ternary semigroup T. Let $a \in T$ such that $a = xyz$ for all $x, y, z \in T$.

Consider $(\mu_1 \circ T \circ \mu_2)(a) = \bigvee_{a=xyz} \{\mu_1(x) \wedge T(y) \wedge \mu_2(z)\}$

$= \bigvee_{a=xyz} \{\mu_1(x) \wedge 1 \wedge \mu_2(z)\}$

$= \bigvee_{a=xyz} \{\mu_1(x) \wedge \mu_2(z)\}$

$\leq \bigvee_{a=xyz} \{\mu_1(xyz) \wedge \mu_2(xyz)\}$

$\leq \mu_1(a) \wedge \mu_2(a)$

$\mu_1 \circ T \circ \mu_2 \subseteq \mu_1 \cap \mu_2$.

If $a \neq xyz$ then $(\mu_1 \circ T \circ \mu_2)(a) = 0 \leq (\mu_1 \cap \mu_2)(a)$

Therefore $\mu_1 \circ T \circ \mu_2 \subseteq \mu_1 \cap \mu_2$.

2.6 Theorem: If a fuzzy subset μ of a ternary semigroup T is a fuzzy quasi ideal in T then the level set μ_t is a quasi ideal of T.

Proof: Let T be a ternary semigroup and given μ_t is a level set of a fuzzy set μ.

Suppose μ is a fuzzy quasi ideal of T and let $a \in [(TT\mu_t) \cap (T\mu_t T) \cap (\mu_t TT)]$. Then there exists $x, y, z \in \mu_t$ $\Rightarrow \mu(x) \geq t$, $\mu(y) \geq t$, $\mu(z) \geq t$. Let $u, v, p, q, r, s \in T$. Then

Consider $[(T \circ T \circ \mu) \cap (T \circ \mu \circ T) \cap (\mu \circ T \circ T)](a)$

$= (T \circ T \circ \mu)(a) \cap (T \circ \mu \circ T)(a) \cap (\mu \circ T \circ T)(a)$

$= [\bigvee_{a=uvx} \{T(u) \wedge T(v) \wedge \mu(x)\}] \wedge [\bigvee_{a=pyq} \{T(p) \wedge \mu(y) \wedge T(q)\}] \wedge [\bigvee_{a=zrs} \{\mu(z) \wedge T(r) \wedge T(s)\}]$

$= [\bigvee_{a=uvx} \{1 \wedge 1 \wedge \mu(x)\}] \wedge [\bigvee_{a=pyq} \{1 \wedge \mu(y) \wedge 1\}] \wedge [\bigvee_{a=zrs} \{\mu(z) \wedge 1 \wedge 1\}]$

$= [\bigvee_{a=uvx} \{\mu(x)\}] \wedge [\bigvee_{a=pyq} \{\mu(y)\}] \wedge [\bigvee_{a=zrs} \{\mu(z)\}]$

$\geq t \Rightarrow [(T \circ T \circ \mu) \cap (T \circ \mu \circ T) \cap (\mu \circ T \circ T)](a) \geq t \Rightarrow a \in \mu_t$

Therefore $[(TT\mu_t) \cap (T\mu_t T) \cap (\mu_t TT)] \subseteq \mu_t$

and let $a \in [(TT\mu_t) \cap (TT\mu_t TT) \cap (\mu_t TT)]$. Then there exists $x, y, z \in \mu_t$ and $u, v, p, q, r, s, w \in T$ such that $\mu(x) \geq t$, $\mu(y) \geq t$, $\mu(z) \geq t$.

Consider $[(T \circ T \circ \mu) \cap (T \circ T \circ \mu \circ T \circ T) \cap (\mu \circ T \circ T)](a)$

$= (T \circ T \circ \mu)(a) \wedge (T \circ T \circ \mu \circ T \circ T)(a) \wedge (\mu \circ T \circ T)(a)$

$= [\bigvee_{a=uvx} \{T(u) \wedge T(v) \wedge \mu(x)\}] \wedge [\bigvee_{a=pqr} \{T(p) \wedge T(q) \wedge (\mu \circ T \circ T)(r)\}]$

$\wedge [\bigvee_{a=zsw} \{\mu(z) \wedge T(s) \wedge T(w)\}]$

$= [\bigvee_{a=uvx} \{1 \wedge 1 \wedge \mu(x)\}] \wedge [\bigvee_{a=pqr} \{1 \wedge 1 \wedge (\mu \circ T \circ T)(r)\}] \wedge [\vee_{a=zsw} \{\mu(z) \wedge 1 \wedge 1\}]$

Let $r = ybc$, *for any* $y, b, c \in T$

$= [\bigvee_{a=uvx} \{\mu(x)\}] \wedge [\bigvee_{a=pqr} \{\bigvee_{r=ybc} \{\mu(y) \wedge T(b) \wedge T(c)\}] \wedge [\bigvee_{a=zsw} \{\mu(z)\}]$

$= [\bigvee_{a=uvx} \{\mu(x)\}] \wedge [\bigvee_{a=pqr} \{\bigvee_{r=ybc} \{\mu(y) \wedge 1 \wedge 1\}] \wedge [\bigvee_{a=zsw} \{\mu(z)\}]$

$= [\bigvee_{a=uvx} \{\mu(x)\}] \wedge [\bigvee_{a=pqr} \{\bigvee_{r=ybc} \{\mu(y)\}\}] \wedge [\bigvee_{a=zsw} \{\mu(z)\}]$

$\geq t$

$[(T \circ T \circ \mu) \cap (T \circ T \circ \mu \circ T \circ T) \cap (\mu \circ T \circ T)](a) \geq t \Rightarrow a \in \mu_t$

Therefore $a \in [(TT\mu_t) \cap (TT\mu_t TT) \cap (\mu_t TT)] \subseteq \mu_t$

Hence μ_t is a quasi ideal in T.

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