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# BIANCHI TYPE $VI_0$ MASSIVE STRING COSMOLOGICAL MODELS WITH MAGNETIC FIELD AND TIME DEPENDENT VACUUM ENERGY DENSITY IN GENERAL RELATIVITY

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**ABSTRACT**

Bianchi Type $VI_0$ massive string cosmological models with magnetic field and time dependent vacuum energy density following the technique used by Leterlier [18], are investigated. The conservation equation $(T_i^j - \Lambda g_i^j)_{;j} = 0$ is satisfied for both the models where $\Lambda$ is time dependent vacuum energy density. It is observed that the first model satisfies dominant and weak energy conditions given by Hawking and Ellis [51] while the second model satisfies weak energy conditions. Both the models represent decelerating phase of universe and anisotropy is maintained. In the second model, the vacuum energy density $(\Lambda) \sim \frac{1}{t^2}$ leads to the result obtained by Beesham [41] and State Finder parameters {r,s} agree with $\Lambda$ CDM model. The other physical aspects with singularities in the models, are also discussed. In absence of vacuum energy density ($\Lambda$), the model leads to the model as obtained by Tikekar and Patel [40].



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## INTRODUCTION

The present day universe is satisfactorily described by Friedman-Robertson-Walker models which are homogeneous and isotropic. The universe in smaller scale is neither homogeneous nor isotropic nor do we expect the universe in its early stages to have these properties. Therefore, spatially homogeneous and anisotropic Bianchi line-elements (I – IX) are undertaken to study the universe in its early stages of evolution. Among these, Bianchi Type $VI_0$ cosmological model is of particular interest because this is simple generalization of Bianchi Type I space-time which is the simplest anisotropic and homogeneous model whose spatial sections are flat but the expansion rate is direction dependent. Barrow (Barrow, 1984), in his investigation has pointed out that Bianchi Type $VI_0$ cosmological models give a better explanation of some of the cosmological problems like helium abundances and these can be isotropized in special case. Seeing the importance of these models, several authors viz. Wainwright et al. (Wainwright, 1979). Dunn and Tupper (1978), Collins and Hawking (1973), Ellis and MacCallum (1969), Collins (1971), Roy and Singh (1983), Ram and Singh (1993), Chakraborty (1991), Tikekar and Patel (1994), Bali et al. (2008& 2009) investigated Bianchi Type $VI_0$ cosmological models in different contexts. The study of cosmic strings has created a considerable interest due to the existence of large scale networks of strings in the early universe and is confirmed by the present day observations of the universe (Kibble [13]). These strings arise during the phase transition after the big-bag explosion when the temperature goes down below some critical temperature as predicted by grand unified theories. (Zel'dovich et al. (1980), Everett (1981), Vilenkin (1982). These strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings. The general treatment of strings was initiated by Letelier (1979). Letelier (1979) explained that massive strings are formed by geometric strings with particles attached along its extension. Letelier (1983) first used this idea to find cosmological solutions of massive strings in Bianchi Type I and Kantowski space-times.

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Stachel (1980) developed a classical theory of geometric strings. Several authors viz. Banerjee et al (1990) Wang (2003), Bali and Pradhan (2007), Reddy et al. (Reddy, 2007), Rao and Vinutha (Rao, 2010) have studied homogeneous and anisotropic string cosmological models in different physical and geometrical contexts. The magnetic field plays a significant role at the cosmological scale and is present in galactic and intergalactic spaces. The present day magnitude of magnetic energy is very small in comparison with the estimated matter density and its importance is considered. A cosmological model which contains a global magnetic field is necessarily anisotropic since the magnetic field vector specifies a preferred spatial direction (Bronnikov et al. (Bronnikov, 2004) Melvin (1975) in the cosmological solutions for dust and electromagnetic field, has argued that for a large part of the history of evolution of the universe, the matter was in a highly ionized state and is smoothly coupled with the field and forms a neutral matter as a result of universe expansion. Hence the presence of magnetic field in string-dust universe is not unrealistic. Therefore, several authors viz. Tikekar and Patel (1992), Patel and Maharaj (1996), Singh and Singh (1999), Bali and Anjali (2006), Saha and Visinescu (2008), Saha et al. (2010), Bali (2008), Singh (2014) have investigated string cosmological models with incident magnetic field in different contexts.

A wide range of observations suggest that universe possesses a non-zero cosmological constant (Λ). The cosmological constant (Λ) is the most favoured candidate of dark energy representing energy density of vacuum. Zel'dovich (1968), Dreitlein (1974) Krauss and Turner (1995) have studied its significance from time to time. Recently Barrow and Shaw (2011) suggested that cosmological term corresponds to a very small value of the order $10^{-122}$ when applied to Friedmann universe. A number of cosmological models in which Λ decays with time have been investigated by Berman (1991), Chen and Wu (1990), Beesham (1993), Wang and Meng (2005), Saha (2007 and 2013), Bali and Singh (2008& 2014). A class of solutions in the context of Bianchi $VI_0$ string cosmology has been obtained by Chakraborty (1991), Tikekar and Patel (1994) have obtained some exact solutions in Bianchi $VI_0$ string cosmology with magnetic field. In this paper, we have investigated some Bianchi Type $VI_0$ massive string cosmological models with magnetic field and decaying vacuum energy density. To get the deterministic model of universe, we have assumed that shear (σ) is proportional to expansion (θ) and vacuum energy density $\Lambda \sim \frac{1}{R^3}$ where R is average scale factor. Physical and geometrical aspects of the models are also discussed.

## 2. Metric and Field Equations

We consider Bianchi Type $VI_0$ metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2x} B^2 dy^2 + e^{-2x} C^2 dz^2 \quad \text{......(1)}$$

where A, B, C are metric potentials and are functions of cosmic time t only. If x = 0 then the space-time [1] leads to Bianchi Type I space-time.

The energy momentum tensor of a cloud strings in the presence of magnetic field is taken into the form

$$T_i^j = \rho v_i v^j - \lambda x_i x^j + \left[ -\frac{1}{4} g_i^j F_{\ell m} F^{\ell m} + g^{\ell m} F_{i\ell} F_m^j \right] \quad \text{......(2)}$$

where ρ is the rest energy density with massive particles attached to them and is related to the string tension density (λ) by the relation given by Letelier (1983)
[

$$\rho = \rho_p + \lambda \quad \text{......(3)}$$

$\rho_p$ being the rest energy density of the particles attached to the strings. The matter flow and string's direction are specified by the unit time like $v^i$ and space like $x^i$ vectors satisfying the conditions

$$v_i v^i = -x_i x^i = -1, \; v^i x_i = 0 \quad \text{......(4)}$$

We assume that the string's direction is along x-axis. Thus $x_1 \neq 0$, $x_2 = 0 = x_3$. Thus we have

$$x^i = \left( \frac{1}{A}, 0, 0, 0 \right), \; v^i = (0,0,0,1) \quad \text{......(5)}$$

Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \text{ and } (F^{ij}\sqrt{-g})_{;j} = 0 \quad \text{......(6)}$$

are satisfied by

$F_{23} = K$ (constant) ……………………………….(7)

We assume that current is flowing along x-direction. Thus magnetic field is in yz-plane and $F_{23}$ is the only non-vanishing component of $F_{ij}$.

The Einstein's field equation

$$R_i^j - \frac{1}{2} R\, g_i^j - \Lambda g_i^j = -T_i^j \qquad \text{……………………………….(8)}$$

(in gravitational unit $8\pi G=1, c=1$) for the metric (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{1}{A^2} = -T_1^1 + \Lambda \qquad \text{……………………………….(9)}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = -T_2^2 + \Lambda \qquad \text{……………………………….(10)}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} = -T_3^3 + \Lambda \qquad \text{……………………………….(11)}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = -T_4^4 + \Lambda \qquad \text{……………………………….(12)}$$

$$\frac{1}{A}\left[\frac{C_4}{C} - \frac{B_4}{B}\right] = -T_1^4 \qquad \text{……………………………….(13)}$$

where the components of energy momentum tensor ($T_i^j$) are given by

$$T_1^1 = -\lambda - \frac{K^2}{2B^2C^2} \qquad \text{……………………………….(14)}$$

$$T_2^2 = \frac{K^2}{2B^2C^2} = T_3^3 \qquad \text{……………………………….(15)}$$

$$T_4^4 = -\rho - \frac{K^2}{2B^2C^2} \qquad \text{……………………………….(16)}$$

$$T_1^4 = 0 \qquad \text{……………………………….(17)}$$

Now Einstein's field equation (8) for the metric (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{1}{A^2} = \left(\lambda + \frac{K^2}{2B^2C^2}\right) + \Lambda \qquad \text{……………………………….(18)}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = -\frac{K^2}{2B^2C^2} + \Lambda \qquad \text{……………………………….(19)}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{1}{A^2}=-\frac{K^2}{2B^2C^2}+\Lambda \qquad \text{.........(20)}$$

$$\frac{A_4B_4}{AB}+\frac{B_4C_4}{BC}+\frac{A_4C_4}{AC}-\frac{1}{A^2}=\rho+\frac{K^2}{2B^2C^2}+\Lambda \qquad \text{.........(21)}$$

$$\frac{1}{A}\left(\frac{C_4}{C}-\frac{B_4}{B}\right)=0 \qquad \text{.........(22)}$$

**3. Solution of Field Equations and Methodology**

Equation (22) leads to

$$C = mB \qquad \text{.........(23)}$$

where m is the constant of integration. Thorne [47] has pointed out that Hubble expansion of universe is isotropic within 30%. Thus Red Shift studies place the limit on $\sigma/H \leq 0.30$ where $\sigma$ is shear and H is the Hubble constant. Also as per Collins et al. [48] investigations, $\sigma/\theta$ is constant for spatially homogeneous metric where $\theta$ is the expansion in the model.
Thus, we have

$$A = B^n \qquad \text{.........(24)}$$

where proportionality constant is assumed as unity

and

$$\sigma=\frac{1}{\sqrt{3}}\left(\frac{A_4}{A}-\frac{B_4}{B}\right) \qquad \text{.........(25)}$$

$$\theta=\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C} \qquad \text{.........(26)}$$

To get the deterministic model of the universe, we assume that

$$\Lambda=\frac{\alpha}{R^3}=\frac{\alpha}{ABC} \qquad \text{.........(27)}$$

where $\alpha$ is constant and R is average scale factor.

Using (23) and (24), we have

$$\Lambda=\frac{\alpha}{m\,B^{n+2}} \qquad \text{.........(28)}$$

Now equation (20) with (23), (24) and (28) leads to

$$(n+1)\frac{B_{44}}{B}+n^2\frac{B_4^2}{B^2}=\frac{1}{B^{2n}}-\frac{\ell^2}{B^4}+\frac{\alpha}{mB^{n+2}} \qquad \text{.........(29)}$$

where

$$\ell^2=\frac{K^2}{2m^2} \qquad \text{.........(30)}$$

To get deterministic model of the universe, we assume n = 2. Thus, equation (29) leads to

$$2B_{44}+\frac{8}{3}\frac{B_4^2}{B}=\frac{2N}{3B^3} \qquad \text{.....(31)}$$

where $N=\beta+1-\ell^2$ and $\beta=\frac{\alpha}{m}$. Equation (31) leads to

$$\frac{d}{dB}f^2+\frac{8}{3B}f^2=\frac{2N}{3B^3} \qquad \text{.....(32)}$$

where $B_4=f(B), B_{44}=f\,f', f'=\frac{df}{dB}$.

From equation (32), we hve

$$f^2=\left(\frac{dB}{dt}\right)^2=N\,B^{-2}+L\,B^{-8/3} \qquad \text{.....(33)}$$

where L is constant of integration. Therefore, the metric (1) leads to

$$ds^2=-\left(\frac{dt}{dB}\right)^2 dB^2+B^4dx^2+B^2e^{2x}dy^2+m^2B^2e^{-2x}dz^2 \qquad \text{.....(34)}$$

which leads to

$$ds^2=-\frac{dT^2}{NT^{-2}+LT^{-8/3}}+T^2dX^2+T^2e^{2x}dY^2+m^2T^2e^{-2x}dZ^2 \qquad \text{.....(35)}$$

where cosmic time (t) is defined as

$$t=\int\frac{dT}{\sqrt{NT^{-2}+LT^{-8/3}}} \quad \text{and } B = T. \qquad \text{.....(36)}$$

**4. Special Model**

To get the deterministic model of the universe in terms of cosmic time t, we assume L = 0. Thus equation (33) leads to

$$B^2=at+b \qquad \text{.....(37)}$$

where $a=2\sqrt{N}$ and b is constant of integration. Thus

$$A=B^2=(at+b) \qquad \text{.....(38)}$$

and

$$C^2=m^2B^2=m^2(at+b) \qquad \text{.....(39)}$$

Therefore, the metric (1) leads to the form

$$ds^2=-dt^2+(at+b)^2dx^2+(at+b)e^{2x}dy^2+m^2(at+b)^{-2x}dz^2 \qquad \text{.....(40)}$$

**5. Physical and Geometrical Aspects**

The rest energy density ($\rho$), the string tension density ($\lambda$), the particle density ($\rho_p$), the expansion ($\theta$), the shear ($\sigma$), average scale factor (R). Vacuum energy density ($\Lambda$), the deceleration parameter (q) for the model (35) are given by

$$\rho = (3\beta + 3 - 5\ell^2)T^{-4} + 5L\,T^{-14/3} \qquad \text{.....(41)}$$

$$\lambda = -2\beta\,T^{-4} - \frac{5L}{3}T^{-14/3} \qquad \text{.....(42)}$$

$$\rho_p = \rho - \lambda$$

$$= (5\beta + 3 - 5\ell^2)T^{-4} + \frac{20L}{3}T^{-14/3} \qquad \text{.....(43)}$$

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \qquad \text{.....(44)}$$

$$= 4\frac{B_4}{B} \quad \text{as } A = B^2, C = mB$$

$$= \frac{4}{T^2}\sqrt{N + L\,T^{-2/3}} \qquad \text{.....(45)}$$

$$\sigma = \frac{1}{\sqrt{3}}\left|\frac{A_4}{A} - \frac{B_4}{B}\right|$$

$$= \frac{1}{\sqrt{3}}\left|\frac{B_4}{B}\right| \quad \text{as } A = B^2$$

$$= \frac{1}{\sqrt{3}}\frac{1}{T^2}\sqrt{N + L\,T^{-2/3}} \qquad \text{.....(46)}$$

$$\frac{\sigma}{\theta} = \frac{1}{4\sqrt{3}} \neq 0 \qquad \text{.....(47)}$$

$$R = m^{1/3}T^{4/3} \qquad \text{.....(48)}$$

$$\Lambda = \frac{2}{mT^4} \qquad \text{.....(49)}$$

$$q = -\frac{R_{44}/R}{R_4^2/R^2}$$

$$= \frac{(8NT^{-4} + 12LT^{-14/3})}{16(NT^{-4} + LT^{-14/3})} > 0$$

Conservation equation $(T_i^j - \Lambda g_i^j)_{;j} = 0$

leads to

$$\frac{\partial}{\partial t}(T_4^4) + T_1^1(\Gamma_{12}^2 + \Gamma_{13}^3 + \Gamma_{14}^4) + T_4^4(\Gamma_{14}^1 + \Gamma_{24}^2 + \Gamma_{34}^3)]$$

$$-[T_1^1(\Gamma_{14}^1+T_2^2(\Gamma_{24}^2+\Gamma_{12}^2)+T_3^3(\Gamma_{34}^3+\Gamma_{13}^3)]$$

$$-\Lambda_4=0$$

which leads to

$$\rho_4+\rho\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)-\lambda\frac{A_4}{A}+\Lambda_4=0 \quad \ldots\ldots(50)$$

which again leads to

$$\rho_4+\rho\left(\frac{4B_4}{B}\right)-\lambda\left(\frac{2B_4}{B}\right)+\Lambda_4=0 \quad (\text{as } A=B^2,\ C=mB)$$

Thus, we have

$$-4(3\beta+3-5\ell^2)B^{-4}\frac{B_4}{B}-\frac{70L}{3}B^{-\frac{14}{3}}\frac{B_4}{B}$$

$$+4\,[(3\beta+3-5\ell^2)B^{-4}\frac{B_4}{B}+5L^3B^{-\frac{14}{3}}\frac{B_4}{B}]$$

$$+(4\beta B^{-4}+\frac{10L}{3}B^{-14/3})\frac{B_4}{B}-4\beta B^{-4}\frac{B_4}{B}=0$$

which leads to

$$(-\frac{70L}{3}+20L+\frac{10L}{3})\frac{B_4}{B}B^{-14/3}=0$$

Thus, conservation equation $(T_i^j-\Lambda g_i^j)_{;j}=0$ for the model (35) is satisfied.

The above mentioned quantities for the model (40) are given by

$$\rho=\frac{5a^2}{4(at+b)^2}-\frac{(\beta+1+\ell^2)}{(at+b)^2} \quad \ldots\ldots(51)$$

$$\lambda=-\frac{a^2}{4(at+b)^2}+\frac{1-\ell^2-\beta}{(at+b)^2} \quad \ldots\ldots(52)$$

$$\rho_p=\rho-\lambda$$

$$=\frac{3a^2}{2(at+b)^2}-\frac{2}{(at+b)^2} \quad \ldots\ldots(53)$$

$$\theta=\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}$$

$$=\frac{4B_4}{B}\ \text{as } a=B^2,\ C=mB$$

$$=\frac{2a}{(at+b)} \qquad \ldots\ldots(54)$$

$$\sigma=\frac{1}{\sqrt{3}}\left|\frac{A_4}{A}-\frac{B_4}{B}\right|$$

$$=\frac{1}{\sqrt{3}}\left|\frac{B_4}{B}\right|=\frac{1}{2\sqrt{3}}\frac{a}{(at+b)} \qquad \ldots\ldots(55)$$

$$\frac{\sigma}{\theta}=\frac{1}{4\sqrt{3}}\neq 0 \qquad \ldots\ldots(56)$$

$$\Lambda=\frac{\beta}{(at+b)^2} \qquad \ldots\ldots(57)$$

$$R^3=ABC$$

$$=mB^4$$

$$=m\,(at+b)^2 \qquad \ldots\ldots(58)$$

$$q=-\frac{R_{44}/R}{R_4^2/R^2}$$

$$=\frac{1}{2}>0 \qquad \ldots\ldots(59)$$

Conservation equation $(T_i^j-\Lambda g_i^j)_{;j}=0$ leads to

$$\rho_4+\rho\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)-\lambda\frac{A_4}{A}+\Lambda_4=0 \qquad \ldots\ldots(60)$$

which leads to

$$\rho_4+\rho\left(\frac{4B_4}{B}\right)-\lambda\left(\frac{2B_4}{B}\right)+\Lambda_4=0 \qquad \ldots\ldots(61)$$

which again leads to

$$-\frac{5a^3}{2(at+b)^3}+\frac{2(\beta+1+\ell^2)a}{(at+b)^3}+\left[\frac{5a^2}{4(at+b)^2}-\frac{(\beta+1+\ell^2)}{(at+b)^2}\right]\left(\frac{2a}{at+b}\right)$$

$$-\left[-\frac{a^2}{4(a+b)^2}+\frac{1-\ell^2-\beta}{(at+b)^2}\right]\left(\frac{a}{at+b}\right)-\frac{2\beta a}{(at+b)^3}=0 \qquad \ldots\ldots(62)$$

Thus, we have

$$-\frac{5a^3}{2(at+b)^3}+\frac{5a^3}{2(at+b)^3}+\frac{2(\beta+1+\ell^2)a}{(at+b)^3}-\frac{2(\beta+1+\ell^2)a}{(at+b)^3}$$

$$+\frac{a^3}{4(at+b)^3}-\frac{(1-\ell^2-\beta)a}{(at+b)^3}-\frac{2\beta a}{(at+b)^3}=0$$

which leads to

$$a^2 - 4(1-\ell^2+\beta) = 0 \quad \text{.................................(63)}$$

Thus, we have

$$4N = 4(1-\ell^2+\beta)$$

which is satisfied as $a = 2\sqrt{N}$ and $N = \beta + 1 - \ell^2$.

**State Finder Parameters {r,s}**

The state finder parameter effectively differentiate between forms of dark energy and provide a simple diagnosis whether a particular model fits into the basic observational data. Following Sahni et al. [50], the state finder parameters {r,s} for the model (40) are given by

$$r = 1 + \frac{3\dot{H}}{H^2} + \frac{\ddot{H}}{H^3}$$

$= 1$ where $H = \frac{\theta}{3} = \frac{2a}{3(at+b)}$

and

$$s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} = 0$$

Thus, r = 1, s = 0 agrees with Λ CDM model.

**6. Conclusion**

For the model (35), the expressions for $\rho$, $\rho_p$, $\theta$, $\sigma$ indicate that all the parameters diverge as $T \to 0$. Accordingly, the space-time (35) evolves from the singularity at T = 0 undergoing expansion. The condition $\rho > 0$, $\rho_p > 0$ are satisfied at all epochs. The equation of state are restricted by energy conditions given by Hawking and Ellis [51]. The dominant energy conditions imply that $\rho \geq 0$, and $\rho^2 \geq \lambda^2$. The condition $\rho \geq 0$ leads to

$$(3\beta + 3 - 5\ell^2)T^{2/3} + 5L \geq 0$$

and $\rho^2 \geq \lambda^2$ leads to $(5\beta + 3 - 5\ell^2)T^{2/3} + \frac{20L}{3} \geq 0$

The model starts with a big-bang at T = 0 and expansion decreases with time. The vacuum energy density $\Lambda \alpha \frac{1}{T^4}$. The expressions for $\rho$ and ($\rho_p$) indicate that the magnetic field is directly linked with the matter. For the model (35), the weak energy conditions $\rho \geq 0$, $\lambda < 0$ (Hawking and Ellis [51]) are satisfied. For $\Lambda = 0$, the model leads to the model obtained by Tikekar and Patel [10] and if $\Lambda = 0$, $x = 0$ then the model leads to the model obtained by Banerjee et al. [20]. Since $\frac{\sigma}{\theta} \neq 0$, hence the model represents anisotropic phase of universe. It is also observed that $\rho_p > |\lambda|$ i.e. the particle density remains larger than string tension density during cosmic expansion. The string tension density ($\lambda$) is negative and finally approaches to zero as $T \to \infty$. The deceleration parameter $q > 0$ indicates that the model represents decelerating phase of universe. The model has Point Type Singularity at T = 0 (MacCallum [49]). The conservation equation $(T_i^j - \Lambda g_i^j)_{;j} = 0$ is satisfied for the model (35). The spatial volume increases with time.

For the special model (40), the condition $\rho > 0$ and $\rho_p > 0$ are satisfied at all epochs. The weak energy condition $\rho \geq 0$, $\lambda < 0$ (Hawking and Ellis [51]) are also satisfied. Since $\frac{\sigma}{\theta} \neq 0$, hence anisotropy is maintained throughout. The deceleration parameter $q > 0$ indicates decelerating phase of universe. The model (40) has Point Type Singularity at t = - b/a (MacCallum [49]). The conservation equation $(T_i^j - \Lambda g_i^j)_{;j} = 0$ is satisfied for the model (40) also. For a = 1, b = 0, the vacuum energy density $\Lambda \sim \frac{1}{t^2}$ which leads to the result as obtained by Beesham [41]. It is also observed that $\rho_p > |\lambda|$ i.e. the particle density remains larger than string tension density during cosmic expansion. The state finder parameters {r,s} agrees with Λ CDM model.

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