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SOLUTION OF FIRST ORDER DIFFERENTIAL EQUATION USING NUMERICAL NEWTON'S INTERPOLATION AND LAGRANGE METHOD

***Faith Chelimo Kosgei**

School of Biological and Physical Sciences, Moi University P.O Box 3900 Eldoret Kenya

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ABSTRACT

Differential equation is one of the major areas in mathematics with series of method and solutions. We have analytic method and numerical methods; analytic method is only applicable to a class of equations, so most of the times numerical methods are used. Most of the researches on numerical approach to the solution of first order ordinary differential equation tend to adopt methods such as Runge Kutta method, Taylor series method and Euler's method; but none of the study has actually combined the newt on's interpolation and Lagrange method to solve first order differential equation. This study will combine of Newton's interpolation and Lagrange method to solve the problems of first order differential equation.

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INTRODUCTION

Many problems in real life situation can be formulated in the form of ordinary differential equation, hence the need to solve the differential equations. A numerical method is a tool designed to solve numerical problems. A differential equation as for example u'(x) = Cos(x) for 0 <x< 3 is written as an equation involving some derivative of an unknown function u (Weisstein, 2004). There is also a domain of the differential equation (for the example 0 <x< 3). In reality, a differential equation is then an infinite number of equations, one for each x in the domain. The analytic or exact solution is the functional expression of u or for the example case u(x) = sin(x) + c where c is an arbitrary constant, because of this non uniqueness which is inherent in differential equations we typically include some additional equations. For our example case, an appropriate additional equation would be u(1) = 2 which would allow us to determine c to be 2 − sin(1) and hence recover the unique analytical solution u(x) = sin(x)+2 − sin(1). The differential equation together with the additional equation (s) are denoted a differential equation problem. Note that for our example, if the value of u (1) is changed slightly, for example from 2 to 1.95 then also the values of u are only changing slightly in the entire domain. This is an example of the continuous dependence on data that we shall require: A well-posed differential equation problem consists of at least one differential equation and at least one additional equation such that the system together have one and only one solution (existence and uniqueness) called the analytic or exact solution (Joshn Wiley, 1969) to distinguish it from the approximate numerical solutions that we shall consider. Further, this analytic solution must depend continuously on the data in the (vague) sense that if the equations are changed slightly then also the solution does not change too much. The study in this regard wishes to determine the solution of first order differential equation using combination of Newton's interpolation and Lagrange method.

****Corresponding author:*** **Faith Chelimo Kosgei,**
School of Biological and Physical Sciences, Moi University P.O Box 3900 Eldoret, Kenya.

Literature review

Let's consider the following initial value problem

$$y' = f(x, y) \qquad y(x_0) = y_0$$

Where

f(x, y) is a known function and the values in the initial conditions are also known numbers.

Euler method

The Euler method for approximating the solution to the initial-value problem is given by

$$y_{n+1} = y_n + hf(x_n, y_n)$$

Taylor's series method

The method is given by the expression;

$$y_n = y_{n-1} + hy'_{n-1} + \frac{h^2}{2!}y''_{n-1} + \frac{h^3}{3!}y'''_{n-1} + \cdots$$

Runge kutta method

We do have 1st order Runge kutta, 2nd order Runge kutta 3rd, 4th etc
Runge Kutta – 4

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Therefore the method is given by

$$y_{n+1} = y_n + K$$

Newton's interpolation

$$f_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \cdots\ldots + a_n(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

Where

$$a_0 = y_0$$

$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}$$

$$a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)}$$

Etc.

Lagrange method

$$y_n = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)}y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)}y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}y_2$$

MATERIALS AND METHODS

The study will combine both Newton's interpolation method and Lagrange method to solve first order differential equation. Since the problem is an initial value problem (IVP), the first value for y is available. We will use newton's interpolation to find the second two terms then use the three values for y to form a quadratic equation using Lagrange method as follows;

Newton's interpolation method

$$y_0 = a_0$$

$$y_1 = a_0 + a_1(x - x_0)$$

$$y_1 = a_0 + a_1(x - x_0) + a_1(x - x_0)(x - x_1)$$

Lagrange method

$$y_n = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)}y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)}y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}y_2$$

Example 1

Solve $\frac{dy}{dx} = 1 - y \qquad y(0) = 0$

Taking step h=0.01

Using Newton's interpolation

$$a_0 = 0 = y_0$$

$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = \left[\frac{dy}{dx}\right]_{0,0} = 1$$

$$y_1 = 0 + 1(0.01 - 0) = 0.01$$

$$a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} = \frac{\left[\frac{dy}{dx}\right]_{0.01,0.01} - \left[\frac{dy}{dx}\right]_{0,0}}{0.02 - 0} = -0.5$$

$$y_2 = 0 + 1(0.02 - 0) - 0.5(0.02 - 0)(0.02 - 0.01) = 0.0199$$

Forming quadratic using Lagrange

$$y_n = \frac{(x - 0.01)(x - 0.02)}{(0 - 0.01)(0 - 0.02)} * 0 + \frac{(x - 0)(x - 0.02)}{(0.01 - 0)(0.01 - 0.02)} * 0.01 + \frac{(x - 0)(x - 0.01)}{(0.02 - 0)(0.02 - 0.01)} * 0.0199$$

$$y_n = -0.5x^2 + 1.005x$$

The equation is used to get the values for y at any given value of x

Example 2

Consider the differential equation

$$\frac{dy}{dx} = x^2 - y \qquad y(0) = 1$$

We will take to be h=0.01

$$a_0 = 1$$

$$y_0 = 1$$

$$a_1 = \left[\frac{dy}{dx}\right]_{0,1} = -1$$

$$y_1 = 1 - 1(0.01 - 0) = 0.99$$

$$y_1 = 1 - 1(0.01 - 0) = 0.99$$

$$a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} = \frac{\left[\frac{dy}{dx}\right]_{0.01,0.99} - \left[\frac{dy}{dx}\right]_{0,1}}{0.02 - 0.01} = 0.505$$

$y_2 = 1 - 1(0.02 - 0) + 0.505(0.02 - 0)(0.02 - 0.01) = 0.980101$

Forming quadratic using Lagrange

$$y_n = \frac{(x-0.01)(x-0.02)}{(0-0.01)(0-0.02)} * 1 + \frac{(x-0)(x-0.02)}{(0.01-0)(0.01-0.02)} * 0.99 + \frac{(x-0)(x-0.01)}{(0.02-0)(0.02-0.01)} * 0.980101$$

$$y_n = 0.505x^2 - 1.00505x + 1$$

$$a_2 = \frac{\frac{f(x_2)-f(x_1)}{(x_2-x_1)} - \frac{f(x_1)-f(x_0)}{(x_1-x_0)}}{(x_2-x_0)} = \frac{\left[\frac{dy}{dx}\right]_{0.01,0.505} - \left[\frac{dy}{dx}\right]_{0,0.5}}{0.02-0} = -0.25$$

$y_2 = 0.5 + 0.5(0.02 - 0) - 0.25(0.02 - 0)(0.02 - 0.01) = 0.50995$

Forming quadratic using Lagrange

$$y_n = \frac{(x-0.01)(x-0.02)}{(0-0.01)(0-0.02)} * 0.5 + \frac{(x-0)(x-0.02)}{(0.01-0)(0.01-0.02)} * 0.505 + \frac{(x-0)(x-0.01)}{(0.02-0)(0.02-0.01)} * 0.50995$$

Table 1. The table showing results of the equation $\frac{dy}{dx} = 1 - y$

X	Combined Newton's interpolation and Lagrange	Exact values	Percentage error
0	0	0	0%
0.01	0.01	0.009950166251	0.50083333%
0.02	0.0199	0.019801326	0.498320163%
0.03	0.0297	0.029554466	0.492426423%
0.04	0.0394	0.03921056	0.483135155%
0.05	0.049	0.048770575	0.470468105%
0.06	0.0585	0.058235466	0.454248962%
0.07	0.0679	0.06760618	0.434605238%
0.08	0.0772	0.076883653	0.411461978%
0.09	0.0864	0.086068814	0.384792103%
0.1	0.0955	0.095162581	0.354571093%

Table 2. The table showing results of the equation $\frac{dy}{dx} = x^2 - y$

X	Combined Newton's interpolation and Lagrange	Exact	Percentage error
0	1	1	0%
0.01	0.99	0.990050166	-0.00506701596%
0.02	0.980101	0.980201326	-0.00983363455%
0.03	0.970303	0.970454466	-0.0000015607%
0.04	0.960606	0.96081056	-0.021290357%
0.05	0.950101	0.951270575	-0.12294872%
0.06	0.941515	0.941835466	-0.0340025688%
0.07	0.932121	0.93250618	-0.041305892%
0.08	0.922828	0.923283653	-0.049351355%
0.09	0.913636	0.914168814	-0.058283983%
0.1	0.904545	0.905162582	-0.068228829%

Table 3. The table showing results of the equation $\frac{dy}{dx} = y - x$

X	Combined Newton's interpolation and Lagrange	Exact	Percentage error
0	0.5	0.5	0%
0.01	0.505	0.504974916	0.004967375%
0.02	0.50995	0.50999933	-0.009672561727%
0.03	0.51485	0.514772733	0.015009924%
0.04	0.5197	0.519594612	0.020282735%
0.05	0.5245	0.524364451	0.02585015%
0.06	0.52925	0.529081726	0.031804916%
0.07	0.53395	0.533745909	0.010893273%
0.08	0.5386	0.538356466	0.04523657%
0.09	0.5432	0.542912858	0.05288915%
0.1	0.54775	0.547414541	0.06195205%

Example 3

Solve the initial value problem $y' = y - x \quad y(0) = 0.5$
Taking step h=0.01

$a_0 = 0.5$

$y_0 = 0.5$

$$a_1 = \left[\frac{dy}{dx}\right]_{0,0.5} = 0.5$$

$y_1 = 0.5 + 0.5(0.01 - 0) = 0.505$

$$y_n = -0.25x^2 + 0.5025x + 0.5$$

RESULTS AND DISCUSSION

The method that has been used gives results very close to the exact value. This is noted by the percentage error that is very minor. The method is very accurate and easy to use after etting the quadratic equation. Thus one can get the value of y at any value of x without necessary getting preceding values of y.

Conclusion and Recommendation

Numerical methods used such as Runge kutta, Euler, Taylor series methods etc. are cumbersome and have a bigger percentage error. I therefore recommend the use of combined Newton's interpolation and Lagrange method to solve first order differential equation.

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