Available online at http://www.journalijdr.com

IJDR

ISSN: 2230-9926

International Journal of Development Research
Vol. 09, Issue, 11, pp. 31065-31069, November, 2019

RESEARCH ARTICLE OPEN ACCESS

MATHEMATICAL MODELING OF STATIONARY TEMPERATURE FIELDS FOR CONSTRUCTION OF MICROELECTRONIC DEVICES

***Yordan L. Milev and Marin H. Hristov**

Department of Semiconductor and Microelectronic Technologies, Technical University of Sofia

ARTICLE INFO

Article History:
Received 18th August, 2019
Received in revised form
11th September, 2019
Accepted 19th October, 2019
Published online 20th November, 2019

Key Words:
Staionary temperature fields, Microelectronic device, Laplace equation.

ABSTRACT

The problem of analyzing stationary temperature fields in the form of a parallelepiped is under consideration. Heat sources are located on the top of the parallelepiped. The path to the solution of the problem starts from the basic equa-tion of thermal conductivity which is mathematically described with the Laplace equation system.

Citation: Yordan L. Milev and Marin H. Hristov. 2019. "Mathematical modeling of stationary temperature fields for construction of microelectronic devices", *International Journal of Development Research*, 09, (11), 31065-31069.

I. INTRODUCTION

The mathematical model for a multilayer rectangular parallelepiped with the layers of the same size is solved on the basis of physically justified boundary conditions. An algorithm is proposed for calculating the temperature at each point of the examined structure.

II. DESCRIPTION OF THE PROBLEM

$$\frac{\partial^2 T^{(j)}(x,y,z)}{\partial x^2}+\frac{\partial^2 T^{(j)}(x,y,z)}{\partial y^2}+\frac{\partial^2 T^{(j)}(x,y,z)}{\partial z^2}=0 \quad (1)$$

III. ANALYSYS

The analysis is carried out under the following boundary conditions:

$$\frac{\partial T^{(j)}(L_x,y,z)}{\partial x}=0,\frac{\partial T^{(j)}(x,L_y,z)}{\partial y}=0,\frac{\partial T^{(j)}(0,y,z)}{\partial x}=0,\frac{\partial T^{(j)}(x,0,z)}{\partial y}=0, \quad (2)$$

where $j = 1,2,3,\dots,N$- the number of layers in the parallelepiped.

For the first layer the following boundary conditions are fulfilled:

$$-\lambda^{(i)}\frac{\partial T^{(i)}}{\partial u}+\alpha\left(T^{(i)}-T_0\right)=Q(x,y),\ \text{for } z=0, \quad (3)$$

$$Q(x,y) = \begin{cases} \frac{P_i}{l_{x_i} l_{y_i}}; (x_i, y_i) \in D_i \\ 0, (x_i, y_i) \notin D_i. \end{cases}$$

where D_i the domain of the i-th element, l_{xi} and l_{yi} the dimensions of i-th source of heat, $i = 1, 2, \ldots, n_1$; n_1 – number of heat sources, P_i – power dissipated by i-th source; α – coefficient of heat exchange with the environment; $\lambda^{(j)}$ – coefficient of the j-th layer; T_0 – temperature of the environment; $Q(x, y)$ – density function of the heat flow. Let $T_0 = 0$.

In order to ensure the continuity of the heat flow between the layers we introduce the following necessary conditions:

$$T^{(j)}(x, y, z_j) = T^{(j+1)}(x, y, z_j), \tag{4}$$

$$\lambda^{(j)} \frac{\partial T^{(j)}(x,y,z_j)}{\partial z} = \lambda^{(j+1)} \frac{\partial T^{(j)}(x,y,z_j)}{\partial z}, \tag{5}$$

$$T^{(N)}(x, y, z_N) = T_k; (x, y) \in D_L. \tag{6}$$

where:

- D_L is the domain of the base of parallelepiped;
- L_x and L_y – dimensions of the base;
- $T_k = const$;
- $z_N = \sum_{j=1}^{N} h_j$ –) the base points.

IV. SOLUTION

The solution of the problem (1) with the method of Fourier in general form is presented in the form:

$$T(x,y,z) = Z(z).V(x,y) = Z(z)X(x)Y(y).$$

$$X_n^{(j)}(x) = D_n^{(j)}.\cos\frac{n\pi x}{L_x}; Y_m^{(j)}(y) = F_m^{(j)}.\cos\frac{n\pi y}{L_y};$$

$$Z_{nm}^{(j)}(z - z_{j-1}) = H_{nm}^{(j)} sh\gamma_{nm}(z - z_{j-1}) + G_{nm}^{(j)} ch\gamma_{nm}(z - z_{j-1}), \tag{7}$$

$$\gamma_{nm} = \pi\sqrt{\frac{n^2}{L_x^2} + \frac{m^2}{L_y^2}},\ n = 1,2,3,\ldots, m = 1,2,3,\ldots,$$

where n and m are the indexes of the terms of the series; $D_n^{(j)}$, $F_m^{(j)}, H_{nm}^{(j)}, G_{nm}^{(j)}$ are indeterminate constants.

The general solution of the distribution of the temperature field in each layer of the structure with boundary conditions (2)-(6) is assumed in the form:

$$T^{(j)} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} X_n^{(j)}(x)\, Y_n^{(j)}(y) Z_{nm}^{(j)}(z)$$

Substituting the expressions from (7) in the last equality, grouping the indeterminate constants and we get the solution in the form:

$$\begin{aligned} T^{(j)}(x, y, z - z_{j-1}) &= G_{00}^{(j)} + H_{00}^{(j)}(z - z_{j-1}) + \\ &+ \sum_{n=1}^{\infty}\left[H_{n0}^{(j)} sh\frac{n\pi(z - z_{j-1})}{L_x} + G_{n0}^{(j)} ch\frac{n\pi(z - z_{j-1})}{L_x}\right]\cos\frac{n\pi x}{L_x} + \\ &+ \sum_{m=1}^{\infty}\left[H_{0m}^{(j)} sh\frac{m\pi(z - z_{j-1})}{L_y} + G_{0m}^{(j)} ch\frac{m\pi(z - z_{j-1})}{L_y}\right]\cos\frac{m\pi y}{L_y} + \\ &+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[H_{nm}^{(j)} sh\gamma_{nm}(z - z_{j-1}) + G_{nm}^{(j)} ch\gamma_{nm}(z - z_{j-1})\right]\cos\frac{n\pi x}{L_x}\cos\frac{m\pi y}{L_y}. \end{aligned} \tag{8}$$

In order to find coefficients G_{nm} and H_{nm} we use the conditions (3)-(6). We get a system of algebraic equations for coefficients with zero indexes:

$$\left| \begin{array}{c} -\lambda^{(1)}.H_{00}^{(1)} + \alpha G_{00}^{(1)} = A_{00} \\ G_{00}^{(j)} + H_{00}^{(j)} h_j = G_{00}^{(j)} \\ \lambda^{(j)}.H_{00}^{(j)} = \lambda^{(j+1)} H_{00}^{(j+1)} \\ \dots\dots\dots\dots\dots \\ G_{00}^{(N)} + H_{00}^{(N)}.h_N = T_L, j = 1,2,\dots,N \end{array} \right| \quad (9)$$

The system of algebraic equations for the coefficients with the order indices is:

$$\left| \begin{array}{c} -\lambda^{(1)}.\gamma_{nm} H_{nm}^{(1)} + \alpha G_{nm}^{1} = A_{nm} \\ H_{nm}^{(j)} sh\gamma_{nm} h_j + G_{nm}^{(j)} ch\gamma_{nm} h_j = G_{nm}^{(j+1)} \\ \lambda^{(j)} \left[H_{nm}^{(j)} sh\gamma_{nm} h_j + G_{nm}^{(j)} ch\gamma_{nm} h_j \right] = \lambda^{(j+1)} H_{nm}^{(j+1)} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ H_{nm}^{(N)} sh\gamma_{nm} h_N + G_{nm}^{(N)} ch\gamma_{nm} h_N = 0, \end{array} \right| \quad (10)$$

$$n^2 + m^2 \neq 0, j = 1,2,\dots,N.$$

From the boundary condition (3) we determine the constants:

$$A_{nm} = \sum_{i=1}^{n_1} \frac{16P_i}{mn\pi^2 l_{x_i} l_{y_i}} \cos\left(\frac{n\pi x_{0i}}{L_x}\right) \sin\left(\frac{n\pi l_{x_i}}{2L_x}\right) \cos\left(\frac{m\pi y_{0i}}{L_y}\right) \sin\left(\frac{m\pi l_{y_i}}{2L_y}\right)$$

$$A_{n0} = \sum_{i=1}^{n_1} \frac{4P_i}{n\pi l_{x_i} L_y} \cos\left(\frac{n\pi x_{0i}}{L_x}\right) \sin\left(\frac{n\pi l_{x_i}}{2L_x}\right),$$

$$A_{0m} = \sum_{i=1}^{n_1} \frac{4P_i}{m\pi l_{y_i} L_x} \cos\left(\frac{m\pi y_{0i}}{L_y}\right) \sin\left(\frac{m\pi l_{y_i}}{2L_y}\right),$$

$$A_{00} = \frac{1}{L_x L_y} \sum_{i=1}^{n_1} P_i,$$

where x_{0i} and y_{0i} are the coordinates of centers of heat sources.

In order to calculate the function $T^{(j)}(x, y, z - z_j)$ from (8) we have to follow the following algorithm:

1. Calculate the coefficient using the last formula;
2. Using the system (2) find:

$$H_{00}^{(1)} = \frac{\left(T_k - \frac{A_{00}}{\alpha}\right)}{\left(\lambda^{(1)}.S\right)},$$

$$G_{00}^{(1)} = \frac{\lambda^{(1)} H_{00}^{(1)} + A_{00}}{\alpha},$$

where $S = \frac{1}{\alpha} + h - \frac{1}{\lambda_1} + \frac{h_2}{\lambda_2} + \dots + \frac{h_N}{\lambda_N}.$

The recurrent dependence for $H_{00}^{(j)}$ and $G_{00}^{(j)}$ is:

$$\left| \begin{array}{c} H_{00}^{(j)} = \lambda_{j-1}.\frac{H_{00}^{(j-1)}}{\lambda_j} \\ G_{00}^{(j)} = G_{00}^{(j-1)} + h_{j-1}.H_{00}^{(j-1)}, j = 2,3,\dots,N. \end{array} \right|$$

1. Calculate the coefficients A_{n0}, A_{0m} and A_{nm} using the above formulas.
2. From the system (10) we have:

$$\left| \begin{array}{l} G_{nm}^{(N)} = \left(\frac{-c_N}{b_N}\right) H_{nm}^{(N)} = d_N H_{nm}^{(N)} \\ H_{nm}^{(N)} = 1.H_{nm} = l_N H_{nm} \end{array} \right|$$

$$\left| \begin{array}{l} G_{nm}^{(N-1)} = \left(\frac{-b_{N-1}.c_N}{b_N} - \frac{\lambda_N}{\lambda_{N-1}}.c_{N-1}\right) H_{nm}^{(N)} = d_{N-1} H_{nm}^{(N)} \\ H_{nm}^{(N-1)} = \left(\frac{c_{N-1}c_N}{b_N} + \frac{\lambda_N}{\lambda_{N-1}}.b_{N-1}\right).H_{nm}^{(N)} = l_{N-1} H_{nm}^{(N)} \end{array} \right|$$

The recurrent dependence for $G_{nm}^{(j)}$ and $H_{nm}^{(j)}$ is:

$$\left| \begin{array}{l} G_{nm}^{(j)} = \left(b_j.d_{j+1} - \frac{\lambda_{j+1}}{\lambda_j}.c_j.l_{j+1}\right) H_{nm}^{(N)} = d_j H_{nm}^{(N)} \\ H_{nm}^{(j)} = \left(-c_j d_{j+1} + \frac{\lambda_{j+1}}{\lambda_j}.b_j.l_{j+1}\right).H_{nm}^{(N)} = l_j H_{nm}^{(N)}. \end{array} \right| \quad (11)$$

From (11) we determine d_j and l_j for $j = N-2, N-3, \ldots, 1$. Then:

$$\left| \begin{array}{l} H_{nm}^{(N)} = \frac{A_{nm}}{\alpha d_1 - \lambda^{(1)} \gamma_{nm}.l_1}, \\ G_{nm}^{(N)} = d_N.H_{nm}^{(N)}, \end{array} \right|$$

$$\left| \begin{array}{l} G_{nm}^{(j)} = d_j H_{nm}^{(N)}, \\ H_{nm}^{(j)} = l_j.H_{nm}^{(N)}, \end{array} \right| \quad j = 1,2,\ldots,N-1$$

Since

$$\left| \begin{array}{l} b_j = ch(\gamma_{nm}.h_j), \\ c_j = sh(\gamma_{nm}.h_j), \end{array} \right| j = 1,2,\ldots,N.$$

Using the above formulas finite number of coefficients $H_{nm}^{(j)}$ and $G_{nm}^{(j)}$ are calculated.

3. Substitute the coefficients $H_{nm}^{(j)}$ and $G_{nm}^{(j)}$ in series (8). The sum of the known addends of this series represents an approximate temperature value at any point of the multilayer structure.

V. RESUME

The temperature value of any point of the multilayer structure can be used to design the structure of microelectronic devices.

REFERENCES

Bansal, N., Kimbrel, T.and K. Pruhs. Dynamic speed scaling to manage energy and temperature,inProc.45thIEEE Symp.Found.Comput.Sci.(FOCS2004).Washington. DC.2004.

Koval V. A., D.V. Fedasyuk *et al*, Automation of the thermal design of microelectronic devices by the means of the SAPR, Lvov, Wishcha shkola, 1988.

****