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$(QCD)_T$ IS VERY GOOD FOR QUARK – GLUON - PLASMA

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ABSTRACT

We have defined the non-abelian pure gauge theory SU (3) on a torus. Fourier modes are discrete throughout this definition. For enough small size, we have treated the non-glue ball modes as a perturbation of zero modes. Infra-red singularity is not appearing throughout the discrete momentums. The temperature depending contributions of the effective potential of the non-abelian glueball gauge fields are continuously calculated by us, for the first time on an asymmetric torus $L^3 \times \beta$, till the fourth grade of gauge fields. So, L is the length of the torus in space direction and β is the length in time direction (the inverse of temperature). The Phase transition is indicated by the coefficient γ_2' instead of the coupling constant g**.** The critical temperature is $5.6827150752 \times 10^{12}$ K

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INTRODUCTION

The thermodynamic properties of systems of the quantum fields theories are great interest. These systems are described in case of equilibrium by the formalism of the imaginary time which Matsubara introduced. There are active algorithms for the numerical investigation equilibrium problems. The formalism of the real time or Minkowski's space which was introduced to the perturbation theory by relationship with statistic of the non-equilibrium by Schwinger and others is a useful formalism for the calculation of correlation functions depended on time. Treatment of non-equilibrium problems is very important [1-37]. From the point of view of elementary particles physics, these problems must be exposed, a description of the heating of the early universe (according to an available expositing phase) or a description of hadronic material under extreme conditions for studying the experimental results for a short transition to a quark-Gluon-plasma-phase. The third problem is called the anomaly Baryon number violation processes in the stander model. One, principally, can try to treat such problems by the analytical continuation to imaginary time, but in the practice the return of the analytical continuation to real time in many cases, is rarely able to practice in a special case that can not be done when approximations come for use in Euclidean formalism (that can rarely be avoided). The aim of this work is to develop suitable algorithm to describe non-equilibrium processes. The physical background was built by the heating of the early universe, and by a description collision of heavy ions at high energies. The algorithm that is to be developed is based on a combination of the background fields method and one-loop-approximation. This method has been developed for the pure gauge theory (without fermions) with the gauge group theory SU (3). As the effective potential can be calculated in Mincowski's space or in Euclidean-space, the Euclidean formalism was chosen because of plainness. This means that we have calculated the effective potential at finite temperature on the asymmetric torus $L^3 \times \beta$, Meanwhile (L) is the length of the torus in all the three-space direction and β is the length in the time direction.

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The gauge theory is considered on the torus in 1979 by the scientist G.T. Hooft, after that, Lusher [22-23], Van Baal [24-28], J.Kripfganz and C.Michael[29-30], have worked in this field. All these works deal with the glueball spectrum in a small or medium size. Fremionic contributions were considered by J.Kripfganz, C.Michael and Van Baal. The pure gauge theory on the asymmetric torus: $L^3 \times \beta$ was studied and discussed the finite temperature by Al- Chatouri, S.[17] . We followed [17] and [28] when we have calculated the effective potential. That means we have used the one-loop-approximation.

RESEAARCH METHODOLOGY

- Calculation of temperature contributions for the effective potential.
- The investigating about the quark – gluon – plasma phase and determination of the critical temperature $T_{cr.}$

The research method and its materials:

We have mentioned in the introduction that we took the developed numerical algorithm in the Dissertation [17] and the references [22-29] which is based on a combination of the background fields method and one-loop-approximation for the pure gauge theory with the group SU (3). We will follow the reference [17] in all steps.

The gauge theory:

Introduction

In this term, we will discuss the moving of the pure QCD. When the perturbation theory is employed on the QCD theory, it is necessary to use the infra-red cut-off. It's a very kind way which one considers the theory on a torus with d dimensions and puts extreme periodic conditions. These extreme conditions are not allowed to destroy the invariant of the gauge. The gauge potential is periodic till the gauge transformations. We will use the non-local gauge invariant which is introduced in [28]. The modes are divided into glueball and non-glueball. The integration of the non-glueball modes was done by the one-loop-approximation.

The one-loop –approximation

We will only derive from this passage the effective potential at a finite temperature.

The Division into glueball modes and non-glueball modes

We introduce the projector P:

$$PA_\mu = \frac{1}{L^3}\int_{T^3} A_\mu \,, \tag{2.2.1.1}$$

and function of gauge invariant χ :

$$\chi = (1-P)(\partial_\mu A_\mu + i[PA_\mu, A_\mu]) + L^{-1} \times PA_0 \,, \tag{2.2.1.2}$$

with the definition:

$$B_\mu = PA_\mu, Q_\mu = (1-P)A_\mu \tag{2.2.1.3}$$

χ is equivalent to :

$$B_O = 0, \partial_\mu Q_\mu + i[\ B_\mu, Q_\mu\ ] = 0 . \tag{2.2.1.4}$$

One can calculate Faddeev’s - Popov’s determinant to a standard method. Under the infinitesimal gauge transformation.

$\Omega = \exp\left(i\,\varepsilon\,\Lambda\right)$

is:

$$\delta\chi = (1-P)\{D_\mu(PA)D_\mu(A) + i[P(D_\mu(A)),\ A_\mu]\} + L^{-1}\partial_0 P\Lambda + iL^{-1}\times\ \ P[A_0,\Lambda]. \qquad (2.2.1.5)$$

D_μ (A) is the covariant derivative in this relation.

When we divide Λ into $P\Lambda$ and $\Lambda' = (1-P)\Lambda$ we will find:

$$\left[\begin{array}{l}\delta_\Lambda\chi = (1-P)\quad [\ D_\mu(P\Lambda)D_\mu(A)\Lambda' - \lfloor P\lfloor A_\mu,\Lambda'\ ], A_\mu\ \ ]\ ] \\ +\dfrac{1}{L}\partial_0 P\Lambda + \dfrac{i}{L}\times P[A_0,\Lambda'\ ] + [\chi, P\Lambda\ ]\end{array}\right] \qquad (2.2.1.6)$$

The operator M is:

$$M\Lambda = D_\mu(PA)D_\mu(A) + \lfloor A_\mu, P\lfloor A_\mu,\Lambda\ \ ]] \,. \qquad (2.2.1.7)$$

It can express Faddeev-popov's determinant:

$$\Delta(A) = (\int D\Omega\,\delta(\chi^\Omega))^{-1} \qquad (2.2.1.8)$$

$$\Delta(A) = \int D'\psi D'\overline{\psi}\ d\eta d\overline{\eta}\ \ \exp(\ \frac{1}{g_0^2}\ \int Tr(\overline{\psi}\ \mathrm{M}\psi) + Tr(\overline{\eta}\partial_0\eta + \frac{i}{L}\overline{\eta}\times P[A_0,\psi\ \ ])). \qquad (2.2.1.9)$$

ψ and $\overline{\psi}$ are the space sections of the ghost-fields,

the sign $'$ on D means that $\mathrm{P}\psi = \mathrm{P}\overline{\psi} = 0$. While η and $\overline{\eta}$ are constant to the space, it can be explicitly integrated. These integrations about η and $\overline{\eta}$ deliver a constant. This identity (2.2.1.8) can be generalized:

$$\frac{\Delta(A^{\Omega_0})\int D\Omega\delta(\chi - E)}{\int D'E\ \exp[\dfrac{1}{g_0^2\int Tr(E^2)}]} = 1. \qquad (2.2.1.10)$$

Meanwhile, Ω is known throughout $X^{\Omega_0} = \mathrm{E}$ and $'$ means that $\mathrm{PE} = 0$. When we put this in the sum of the states, we conclude that:

$$Z = \frac{\int DA_\mu D'\psi\ D'\overline{\psi}\exp\left[\dfrac{1}{g_0^2}\int(\dfrac{1}{2}Tr(F_{\mu\upsilon}^2(A)) + Tr(E^2) - 2Tr(\overline{\psi}\mathrm{M}\psi)\ \right]}{\int D'E\exp(\dfrac{1}{g_o^2}\int Tr(E^2))}\times\delta(\chi - E) \qquad (2.2.1.11)$$

After doing the integrations about E we conclude the expression of Z:

$$Z = \int DA_\mu\ D'\psi D'\overline{\psi}\exp\left[\frac{1}{g_0^2}\int(\frac{1}{2}Tr(F_{\mu\upsilon}^2(A) + Tr(\chi^2) - 2Tr(\overline{\psi}\mathrm{M}\psi))\ \right]. \qquad (2.2.1.12)$$

From (2.2.1.2) , (2.2.1.3) and(2.2.1.4) we find :

$$\partial_\mu B_\mu = 0 \,. \qquad (2.2.1.13)$$

This leads to:

$$\chi = D_\mu (B) Q_\mu \,. \qquad (2.2.1.14)$$

We put this in (2.2.1.12):

$$Z = \int DB_\mu \; D'\, Q_\mu D'\psi D'\overline{\psi} \exp\left[\frac{1}{g_0^2}\int \frac{1}{2} Tr\left(F_{\mu\upsilon}(B+Q)\right) + Tr\left(\left(D_\mu (B) Q_\mu\right)^2\right) - 2Tr\left(\overline{\psi} D_\mu (B) D_\mu (B+Q)\psi\right) - 2Tr\left(\left[Q_\mu, \psi\right] p \left[Q_\mu, \overline{\psi}\right]\right)\right] \qquad (2.2.1.15).$$

one can simply derive effective Lagrange function for B.

$$Z = \int DB_\kappa \exp(\int d\tau L_{eff}(B)) = \int DB_\kappa \exp(S_{eff}) \,. \qquad (2.2.1.16)$$

This means:

$$S_{eff} = \int d\tau \; L_{eff}(B) = \log \int D'Q_\mu D'\psi D'\overline{\psi} \exp(\frac{1}{g_0^2}\int d\tau \int d^3 x L(B, Q, \psi, \overline{\psi})) \qquad (2.2.1.17)$$

Meanwhile, $L(B, Q, \psi, \overline{\psi})$ will take the following form:

$$L(B, Q, \psi, \overline{\psi}) = Tr\,(\frac{1}{2}(F_{\mu\nu}(B+Q))^2 + (D_\mu (B) Q_\mu)^2 - 2\overline{\psi} D_\mu (B) D_\mu (B+Q)\psi - 2[Q_\mu, \psi] P [Q_\mu, \overline{\psi}] \;). \qquad (2.2.1.18)$$

When we develop $[F_{\mu\nu}(B+Q)]^2$ till the second grade of Q, we get:

$$\int \frac{1}{2} Tr((F_{\mu\nu}(B+Q))^2) = \int (\frac{1}{2} Tr(F_{ij}^2 (B)) + Tr(Q_\mu W_{\mu\nu} Q_\nu) - Tr((D_\mu Q_\mu)^2)) \,. \qquad (2.2.1.19)$$

So, it is:

$$W_{\mu\nu} Q_\nu = -D_\nu^2 (B) Q_\mu - 2i\left[F_{\mu\nu}, Q_\nu\right].$$

When we put this in $L(B, Q, \psi, \overline{\psi})$ and take terms till the second grade of Q, ψ and $\overline{\psi}$ we get:

$$L(B, Q, \psi, \overline{\psi}) = Tr(\frac{1}{2} F_{\mu\nu}^2 (B)) + Tr(Q_\mu W_{\mu\nu} Q_\nu) - 2Tr(\overline{\psi} D_\mu^2 (B)\psi) \qquad (2.2.1.20)$$

From (2.2.1.17) and (2.2.1. 20), we get:

$$\int_0^\tau d\tau L_{eff}(B) = -\log \int D'\, Q_\mu D'\psi D'\overline{\psi} \exp[-\frac{1}{g_0^2}\int_0^\tau d\tau \int_{T^3} d^3 x (Tr(\frac{1}{2} F_{\mu\nu}^2 (B)) + Tr(Q_\mu W_{\mu\nu} Q_\nu) - 2Tr(\overline{\psi} D_\mu^2 (B)\psi))]. \qquad (2.2.1.21)$$

Integrations on $\psi, \overline{\psi}, Q$ are Gauss integrations and supply a determinant. After that, we get the expression of the effective potential:

$$\int_0^\tau d\tau\, V_{eff(1)} = -1og[\frac{\det'(-D_\mu^2(B))}{(\det' W_{\mu\nu}(B))^{\frac{1}{2}}}] \ . \qquad (2.2.1.22)$$

The index (1) is to one-loop-approximation, so $D_\mu(B)$ is inverse ghost-propagator and:

$$W_{\mu\nu}(B) = -\delta_{\mu\nu} D^2(B) - 2iadF_{ij}(B), \qquad (2.2.1.23)$$

the propagation of the inverse vector propagator. $adF_{ij}(B)$ is $F_{ij}(B)$in the adjoint representation which is known in the appendix C.

$$D^2 = \partial^2 + 2iadB_i\partial_i - (adB_i)^2 \qquad (2.2.1.24)$$

So, adB_i is the vector potential B_i in the adjoint representation.

In the momentum representation , it confirms :

$$\boldsymbol{D^2 = -K^2 - 2adB_iK_i - (adB_i)^2}. \qquad (2.2.1.25)$$

The equation (2.2.1. 22) is written as :

$$\int_0^\tau d\tau V_{eff(1)} = -1og\ \det'(-D_\mu^2(B)) + \frac{1}{2}1og\ \det' W_{\mu\nu}(B) \qquad (2.2.1.26)$$

Development with the grades of B

In order to calculate both the determinants , we have to use the following identity:

$$\begin{aligned}\log\det(A+C) &= Tr\ \log(A+C) = Tr\ \log A + Tr\ \log(1+CA^{-1})\\ &= Tr\ \log A - \sum_{n=1}\frac{(-1)^n}{n}Tr((CA^{-1})^n).\end{aligned} \qquad (2.2.2.1)$$

In order to calculate $\left(-\frac{1}{2}\log\det W_{\mu\nu}(B)\right)$, (2.2.2.1) is written as :

$$\frac{1}{2}\log\det`(A+C) = \frac{1}{2}Tr\log A + \frac{1}{2}Tr(CA^{-1}) - \frac{1}{4}Tr((CA^{-1})^2) + \frac{1}{6}Tr((CA^{-1})^3) - \frac{1}{8}Tr((CA^{-1})^4)\ . \qquad (2.2.2.2)$$

Meanwhile, it is:

$$A = -\delta_{\mu\nu}\partial^2 \qquad (2.2.2.3)$$

And:

$$C = -\delta_{\eta\nu}(2iadB_i\partial_i - (adB_i)^2) - 2iadF_{ij}(B). \tag{2.2.2.4}$$

This means that we are calculating the determinant till the forth grade of B_i^a . We introduce Fourier transformations:

$$A^{-1} = A^{-1}(x,x') = \frac{1}{(2\pi)^{d+1}} \sum_{k_0} \sum_{\vec{k}\neq\vec{0}} \frac{\delta_{\mu\upsilon} \exp[ik(x-x')]}{k_0^2 + |\vec{k}|^2}$$

$$CA^{-1}(x,x') = \frac{1}{(2\pi)^{d+1}} \sum_{k_0} \sum_{\vec{k}\rangle\neq\vec{0}\rangle} \frac{\exp[ik(x-x')]}{k_0^2 + |\vec{k}|^2} \left[2adB_ik_i + (adB_i)^2 \;)\delta_{\mu\upsilon} - 2iadF_{ij}(B)\right]. \tag{2.2.2.5}$$

Now, we calculate the trace on space – time:

$$\frac{1}{2} Tr\left(CA^{-1}\right) = \frac{(1+d)}{2(2\pi)^{d+1}} \int d^dx \sum_{k_0} \sum_{\vec{k}\neq\vec{0}} \frac{1}{k_0^2 + |\vec{k}|^2} Tr\left((adB_i)^2\right)$$

$$-\frac{1}{4} Tr\left(\left(CA^{-1}\right)^2\right) = -\frac{1}{(2\pi)^{d+1}} \int d^dx \sum_{k_0} \sum_{\vec{k}\neq\vec{0}} \left[\frac{(1+d)k_ik_j}{\left(k_0^2 + |\vec{k}|^2\right)^2} \times\right.$$

$$Tr\left((adB_i)(adB_j)\right) + \frac{1+d}{4} \frac{1}{\left(k_0^2 + |\vec{k}|^2\right)^2} Tr\left((adB_i)^2 \times\right.$$

$$\left.\left.(adB_j)^2\right) + \frac{1}{\left(k_0^2 + |\vec{k}|^2\right)^2} Tr\left((adF_{ij}(B))^2\right)\right]$$

$$\frac{1}{6} Tr\left(\left(CA^{-1}\right)^3\right) = \frac{1}{(2\pi)^{d+1}} \int d^dx \sum_{k_0} \sum_{\vec{k}\neq\vec{0}} \frac{2(1+d)}{d} \frac{|\vec{k}|^2}{\left(k_0^2 + |\vec{k}|^2\right)^3} \times$$

$$Tr\left((adB_i)^2 (adB_j)^2\right)$$

$$-\frac{1}{8} Tr\left(\left(CA^{-1}\right)^4\right) = \frac{-1}{(2\pi)^{d+1}} \int d^dx \sum_{k_0} \sum_{\vec{k}\neq\vec{0}} 2(d+1) \frac{k_i\, k_j k_k k_\ell}{\left(k_0 + |\vec{k}|^2\right)^4} \times$$

$$Tr\left(adB_i\, adB_j\, adB_k\, adB_\ell\right). \tag{*}$$

We use the same identity to calculate the other determinants

$$-\log \det'\left(-D^2\right) = \log \det'\left(A' + C'\right) = Tr \log\left(A' + C'\right) = Tr \log A'$$

$$-\sum_{n=1} \frac{(-1)^n}{n} Tr\left(\left(C'A'^{-1}\right)^n\right)$$

$$= -Tr \log A' + Tr\left(C'A'^{-1}\right) + \frac{1}{2} Tr\left(\left(C'A'^{-1}\right)^2\right)$$

$$-\frac{1}{3}Tr\left(\left(C'A'^{-1}\right)^3\right)+\frac{1}{4}Tr\left(\left(C'A'^{-1}\right)^4\right). \qquad (2.2.2.6)$$

It is by this:

$$A' = -\partial^2$$

$$C' = 2\,iadB_i + \left(adB_i\right)^2$$

$$C'A'^{-1}(x',x') = \frac{1}{(2\pi)^{d+1}}\sum_{k_0}\sum_{\vec{k}\neq\vec{0}}\frac{\exp\left(ik\left(x-x'\right)\right)}{k_0^2+\left|\vec{k}\right|^2}\left(adB_i k_i + \left(adB_j\right)^2\right).$$

While calculating the trace on space. time, we get the following equations:

$$-Tr\left(C'A'^{-1}\right) = -\frac{1}{(2\pi)^{d+1}}\int d^{d+1}x\sum_{k_0}\sum_{\vec{k}\neq\vec{0}}\frac{1}{k_0^2+\left|\vec{k}\right|^2}Tr\left(\left(adB_j\right)\right)^2$$

$$\frac{1}{2}Tr\left(\left(C'A'^{-1}\right)^2\right) = \frac{1}{(2\pi)^{d+1}}\int d^{d+1}x\sum_{k_0}\sum_{\vec{k}\neq\vec{0}}\left[\frac{2K_ik_j}{\left(k_0^2+\left|\vec{k}\right|^2\right)^2}\times\right.$$

$$\left. Tr\left(\left(adB_i\right)\left(adB_j\right)\right)+\frac{1}{2}\frac{1}{\left(k_0^2+\left|\vec{k}\right|^2\right)^2}Tr\left(\left(adB_i\right)^2\times\left(adB_j\right)^2\right)\right]$$

$$-\frac{1}{3}Tr\left(\left(C'A'^{-1}\right)^3\right) = \frac{1}{(2\pi)^{d+1}}\int d^{d+1}x\sum_{k_0}\sum_{\vec{k}\neq\vec{0}}\frac{4}{d}\frac{\left|\vec{k}\right|^2}{\left(k_0^2+\left|\vec{k}\right|^2\right)^3}\times$$

$$Tr\left(\left(adB_i\right)^2\left(adB_j\right)^2\right).$$

$$\frac{1}{4}Tr\left(\left(C'A'^{-1}\right)^4\right) = \frac{1}{(2\pi)^{d+1}}\int d^{d+1}x\sum_{k_0}\sum_{\vec{k}\neq\vec{0}}\frac{4k_ik_jk_kk_1}{\left(k_0^2+\left|\vec{k}\right|^2\right)^4}\times Tr\left(adB_i adB_j adB_k adB_\ell\right). \qquad (**)$$

We put (*) in (2.2.2.5) and (**) in (2.2.2.6) , then we get the following equations :

$$\frac{1}{2}\log\det{}'W_{\mu\nu}(B) = \frac{1}{2}\log\det\left(A+C\right)$$

$$= \frac{1+d}{2}\frac{1}{(2\pi)^{d+1}}\int d^{d+1}x\sum_{k_0}\sum_{\vec{k}\neq\vec{0}}\frac{1}{k_0^2+\left|\vec{k}\right|^2}\times$$

$$Tr\left(\left(adB_i\right)^2\right)-\frac{1}{(2\pi)^{d+1}}\int d^{d+1}x\sum_{k_0}\sum_{\vec{k}\neq\vec{0}}\Bigg[$$

$$\frac{(1+d)k_ik_j}{\left(k_0^2+\left|\vec{k}\right|^2\right)^2}Tr\left(\left(adB_i\right)\left(adB_j\right)\right)-\frac{(1+d)k_ik_j}{4\left(k_0^2+\left|\vec{k}\right|^2\right)^2}\times$$

$$Tr\left(\left(adB_{i}\right)^{2}\left(adB_{j}\right)^{2}\right)+\frac{1}{\left(k_{0}^{2}+\left|\vec{k}\right|^{2}\right)^{2}}\times$$

$$Tr\left(\left(adF_{ij}(B)\right)\right)\Big]+\frac{1}{(2\pi)^{d+1}}\int d^{d+1}x\sum_{k_0}\sum_{\vec{k}\neq\vec{0}}\Bigg($$

$$\frac{2(1+d)}{d}\cdot\frac{\left|\vec{k}\right|^{2}}{\left(k_{0}^{2}+\left|\vec{k}\right|^{2}\right)^{3}}Tr\left(\left(adB_{i}\right)^{2}\left(adB_{j}\right)^{2}\right)\Bigg)$$

$$-\frac{1}{(2\pi)^{d+1}}\int d^{d+1}x\sum_{k_0}\sum_{\vec{k}\neq 0}2(1+d)\frac{k_i k_j k_k k_\ell}{\left(k_{0}^{2}+\left|\vec{k}\right|^{2}\right)^{4}}\times$$

$$Tr\left(adB_i adB_j adB_k adB_\ell\right)+o\left(B^{6}\right). \tag{2.2.2.7}$$

$$-\log\ \det'\left(-D_{\mu}^{2}(B)\right)=-\log\ \det'\left(A'+C'\right)$$

$$=-\frac{1}{(2\pi)^{d+1}}\int d^{d+1}x\sum_{k_0}\sum_{\vec{k}\neq\vec{0}}\frac{1}{k_{0}^{2}+\left|\vec{k}\right|^{2}}\times$$

$$Tr\left(\left(adB_i\right)^2\right)+\frac{1}{(2\pi)^{d+1}}\int d^{d+1}x\sum_{K_0}\sum_{K\neq\vec{0}}\left[\frac{2k_i k_J}{(k_0^2+\left|\vec{k}\right|^2)^2}Tr((adB_i)(adB_j))+\frac{1}{2(k_0^2+\left|\vec{k}\right|^2)^2}\times\right.$$

$$\left.Tr((adB_i)^2(adB_j)^2)\right]-\frac{1}{(2\pi)^{d+1}}\times$$

$$\int d^{d+1}x\sum_{k_0}\sum_{\vec{k}\neq\vec{0}}\frac{4\left|\vec{k}\right|^2}{d(k_0^2+\left|\vec{k}\right|^2)^3}Tr((adB_i)^2(adB_j)^2)+\frac{1}{(2\pi)^{d+1}}\times$$

$$\int d^{d+1}x\sum_{k_0}\sum_{\vec{k}\neq\vec{0}}\frac{4k_i k_j k_k k_1}{(k_0^2+\left|\vec{k}\right|^2)^4}Tr(adB_i\ adB_j\times$$

$$adB_k\ adB_\ell)+o(B^6). \tag{2.2.2.8}$$

when we put (2.2.2.7) and (2.2.2.8) in (2.2.1.27) , then we get – for the effective potential – the following expression

$$v_{eff(1)}=\frac{1}{(2\pi)^{d+1}}\int d^{d}x\left[\left[\frac{d-1}{2}\sum_{k_0}\sum_{\vec{k}\neq\vec{0}}\frac{1}{k_0^2+\left|\vec{k}\right|^2}+(1-d)\times\sum_{k_0}\sum_{\vec{k}\neq\vec{0}}\frac{\left|\vec{k}\right|^2}{(k_0^2+\left|\vec{k}\right|^2)^2}\right]\times\right.$$

$$Tr((adB_i)^2)+\left[\frac{(2\pi)^{d+1}}{8g_0^2}-\sum_{k_0}\sum_{\vec{k}\neq 0}\frac{1}{\left(k_0^2+\left|\vec{k}\right|\right)^2}\right]Tr\left(\left(adF_{ij}(B)\right)^2\right)-$$

$$\left[\frac{d-1}{4}\sum_{K_0}\sum_{\vec{K}\neq\vec{0}}\frac{1}{\left(k_0^2+\left|\vec{k}\right|\right)^2}+\frac{2(d-1)}{d}\sum_{K_0}\sum_{\vec{K}\neq\vec{0}}\frac{\left|\vec{k}\right|^2}{(k_0^2+\left|\vec{k}\right|^2)^3}\right]\times$$

$$Tr\left((adB_i)^2(adB_j)^2\right)-2(d-1)\times$$

$$\left.\sum_{K_0}\sum_{\vec{K}\neq\vec{0}}\frac{K_iK_jK_KK_\ell}{(k_0^2+\left|\vec{k}\right|^2)^4}Tr\left(adB_i\; adB_j\; adB_K\; adB_\ell\right)\right] .$$

(2.2.2.9)

The case of the vanish temperature

The sum $\sum_{K_0}$ is considered integration on K_0. After doing the integration on K_0, the effective potential of one-loop-approximate will take the following expression:

$$V_{eff(1)}=\gamma_1\hat{B}_i^a\hat{B}_i^a+\frac{1}{4}\left(\frac{1}{g^2(L)}+\gamma_2\right)\left(f^{abc}\hat{B}_i^b\hat{B}_j^c\right)^2$$

$$+\gamma_3S^{abcd}\hat{B}_i^a\hat{B}_i^b\hat{B}_j^c\hat{B}_j^d+\gamma_4S^{abcd}\hat{B}_i^a\hat{B}_i^b\hat{B}_i^c\hat{B}_i^d \qquad (2.2.3.1)$$

Meanwhile, the coefficients are

$$\gamma_1=\frac{1}{(2\pi)^d}\int d^dx\left[\frac{3(d-1)^2}{4d}\sum_{\vec{K}\neq\vec{0}}\frac{1}{\left|\vec{K}\right|}\right]$$

(2.2.3.2)

$$\gamma_2=\frac{1}{(2\pi)^d}\int d^dx\left[-\frac{d^2+17d+6}{8d}\sum_{\vec{K}\neq\vec{0}}\frac{1}{\left|\vec{K}\right|^3}\right]$$

(2.2.3.3)

$$\gamma_3=\frac{1}{(2\pi)^d}\int d^dx\left[\frac{(d-1)}{16d}\sum_{\vec{K}\neq\vec{0}}\left|\vec{K}\right|^{-7}\left[(6-d)\left|\vec{K}\right|^4-15dk_1^2k_2^2\right]\right]$$

(2.2.3.4)

$$\gamma_4=-\frac{5(d-1)}{16}\sum_{\vec{K}\neq\vec{0}}\frac{\left(k_1^4-3k_1^2k_2^2\right)}{\left|\vec{k}\right|^7} .$$

(2.2.3.5)

This conclusion accords to the reference [28].

The case of the non- vanish temperature From (2.2.2.9) results :

$$V_{eff(1)}=\gamma_1'\hat{B}_i^a\hat{B}_i^a+\frac{1}{4}\left(\frac{1}{g^2(L)}+\gamma_2'\right)\left(f^{abc}\hat{B}_i^b\hat{B}_j^c\right)^2$$

(2.2.4.1)

$$+\gamma_3'S^{abcd}\hat{B}_i^a\hat{B}_i^b\hat{B}_j^c\hat{B}_j^d+\gamma_4'S^{abcd}\hat{B}_i^a\hat{B}_i^b\hat{B}_i^c\hat{B}_i^d$$

So, the coefficients:

$$\gamma_1' = \frac{1}{(2\pi)^{d+1}}\int d^d x\left[\frac{3(d-1)}{2}\sum_{K_0}\sum_{\vec{K}\neq\vec{0}}\frac{1}{k_0^2+|\vec{k}|^2}+\right.$$ (2.2.4.2)

$$\left.3(1-d)\sum_{K_0}\sum_{\vec{K}\neq\vec{0}}\frac{|\vec{k}|^2}{\left(k_0^2+|\vec{k}|^2\right)^2}\right].$$

$$\gamma_2' = \frac{1}{(2\pi)^{d+1}}\int d^d x\left[-\frac{(d+23)}{2}\sum_{K_0}\sum_{\vec{K}\neq\vec{0}}\frac{1}{k_0^2+|\vec{k}|^2}-\frac{8(1-d)}{2d}\times\right.$$

$$\left.\sum_{K_0}\sum_{\vec{K}\neq\vec{0}}\frac{|\vec{k}|^2}{\left(k_0^2+|\vec{k}|^2\right)^3}\right]$$ (2.2.4.3)

$$\gamma_3' = \frac{-1}{(2\pi)^{d+1}}\int d^d x\left[\frac{3(d-1)}{8}\sum_{K_0}\sum_{\vec{K}\neq\vec{0}}\frac{1}{\left(k_0^2+|\vec{k}|^2\right)^2}+\frac{3(1-d)}{d}\sum_{K_0}\sum_{\vec{K}\neq\vec{0}}\frac{|\vec{k}|^2}{\left(k_0^2+|\vec{k}|^2\right)^3}\right]$$ (2.2.4.4)

$$\gamma_4' = \frac{1}{(2\pi)^{d+1}}\int d^d x\left[-(d-1)\sum_{K_0}\sum_{\vec{K}\neq\vec{0}}\frac{k_1^4-3k_1^2k_2^2}{\left(k_0^2+|\vec{k}|^2\right)^4}\right].$$

(2.2.4.5)

One can calculate these coefficients by the helping of the heat kernel. Up from now, we will omit $\int d^d x$ because this integration delivers only the constant L^3 . The definition of the kernels g_1 and g_2 , which appear in the calculation is that one can find in the appendix A. We will divide the coefficients into: related to heat parts and others are not so. By this, we can write $v_{eff(1)}$ as:

$$V_{eff(1)} = V_{eff}^0 + V_{eff}^T$$

So, $V_{eff(1)}^0$ is the unrelated to heat part and $V_{eff(1)}^T$ is the one which is related to heat.

From (2.2.4.2) , (B.7) and (B. 8) we result to :

$$\gamma_1' = \frac{3(d-1)}{2\beta L^d\Gamma(1)}\int_0^\infty dt\ g_1\left(g_2^d-1\right)+\frac{3(1-d)(-1)}{\beta L^d\Gamma(2)}\int_0^\infty dt\ tg_1\ g_2' g_2^{d-1}\ .$$ (2.2.4.6)

Then, we put (A. 12) in (2.2.4.6) :

$$\gamma_1' = \frac{3}{2}(d-1)\left[\frac{1}{\beta L^d}\int_0^\infty dt\left[\frac{\beta}{\sqrt{4\pi}}t^{-\frac{1}{2}}+\frac{\beta}{\sqrt{\pi}}t^{-\frac{1}{2}}\sum_{n_0=1}^\infty \exp(-\frac{\beta^2}{4t}n_0^2)\right](g_2^d-1)\right]$$

$$+3(1-d)(-1)\left[+\frac{1}{\beta L^d}\int_0^\infty dt\ t(\frac{\beta}{\sqrt{4\pi}}t^{-\frac{1}{2}}+\frac{\beta}{\sqrt{\pi}}t^{-\frac{1}{2}}\sum_{n_0=1}^\infty \exp(-\frac{\beta^2}{4t}n_0^2)g_2'\times g_2^{d-1}\right].$$

(2.2.4.7)

At the end, γ_1 becomes into two parts: one is related to heat $\gamma_1'(T \neq 0)$ and other which is not related to heat γ_1 :

$$\gamma_1 = \gamma_1 + \gamma_1'(T \neq O) \quad . \tag{2.2.4.8}$$

So, it is:

$$\gamma_1 = -\frac{3}{V\pi L^3}\bar{t}^{\frac{1}{2}} - \frac{3}{8\pi^2}\int_0^{\bar{t}} dt\, t^{-3} h_2' \;\; h_2^2 + \frac{3}{2\sqrt{\pi}L^3}\int_{\bar{t}}^{\infty} dt\, t^{-\frac{1}{2}}\left(g_2^3 - 1\right) + \frac{3}{V\pi L^3}\int_{\bar{t}}^{\infty} dt\, t^{\frac{1}{2}} g_2' \;\; g_2^2$$

$$\gamma_1' \;(t \neq o) = -\frac{3}{\sqrt{\pi}\;L^3}\int_0^{\bar{t}} dt\; t^{-\frac{1}{2}} \;\; h - \frac{3}{4\pi^2}\int_0^{\bar{t}} \; dt\;\; t^{-3} h\;\, h_2' \;\;\; h_2^2 + \frac{3}{\sqrt{\pi}\;L^3}\times$$

$$\int_{\bar{t}}^{\infty} dt\;\; t^{-\frac{1}{2}} \;\; h\;\left(g_2^3 - 1\right) + \frac{6}{\sqrt{\pi}\;L^3}\int_{\bar{t}}^{\infty} dt\;\; t^{\frac{1}{2}} \;\; h\;\;\; g_2' \;\;\; g_2^2 \;\; . \tag{2.2.4.9}$$

From (2.2.4.3) , (B.7) and (B.8) results :

$$\gamma_2' = \frac{-\left(d + 23\right)}{2\beta L^d\;\Gamma(2)}\int_0^{\infty} dt \quad t\;\, g_1\left(g_2^d - 1\right) - \frac{4\left(1 - d\right)\left(-d\right)}{d\beta L^d\,\Gamma(3)}\int_0^{\infty} dt\; t^2 \;\;\; g_1 \;\;\; g_2' \;\;\; g_2^{d-1} \;\; . \tag{2.2.4.10}$$

We put, after that, (A.12) in (2.2.4.10) and find:

$$\gamma_2' = -\frac{\left(d + 23\right)}{2}\left[\frac{1}{2\sqrt{\pi}\,L^d}\int_0^{\infty} dt\;\; t^{\frac{1}{2}}\left(g_2^d - 1\right)\right] -$$

$$\frac{4\left(1 - d\right)}{d}\left[\frac{-d}{4\sqrt{\pi}\;\;L^d}\int_0^{\infty} dt\;\; t^{\frac{3}{2}} g_2'\; g_2^{d-1}\right] -$$

$$\frac{\left(d + 23\right)}{2}\left[\frac{1}{\sqrt{\pi}\;\;L^d}\int_0^{\infty} dt\;\;\; t^{\frac{1}{2}}\sum_{n_0=1}^{\infty} \exp\;\left(-\frac{\beta^2}{4t}n_0^2\right)\left(g_2^d - 1\right)\right] -$$

$$\frac{4\left(1 - d\right)}{d}\left[\frac{-d}{2\sqrt{\pi}\;\;L^d}\int_0^{\infty} dt\;\; t^{\frac{3}{2}}\sum_{n_0=1}^{\infty} \exp\;\left(-\frac{\beta^2}{4t}n_0^2\right) g_2' g_2^{d-1}\right] . \tag{2.2.4.11}$$

This means:

$$\gamma_2' = \gamma_2 + \gamma_2'(T \neq 0) \;, \tag{2.2.4.12}$$

that:

$$\gamma_2 = -\frac{\left(d + 23\right)}{2}\left[\frac{1}{2\sqrt{\pi}\;\;L^d}\int_0^{\infty} dt\; t^{\frac{1}{2}}\left(g_2^d - 1\right)\right] - \frac{4\left(1 - d\right)}{d}\times \left[\frac{-d}{4\sqrt{\pi}\;L^d}\times\int_0^{\infty} dt\; t^{\frac{3}{2}} \;\; g_2' \;\; g_2^{d-1}\right] \tag{2.2.4.13}$$

and:

$$\gamma_2'(T \neq 0) = -\frac{\left(d + 23\right)}{2}\left[\frac{1}{\sqrt{\pi}\;\;L^d}\int_0^{\infty} dt\; t^{\frac{1}{2}}\sum_{n_0=1}^{\infty}\exp\left(-\frac{\beta}{4t}\,n_0^2\right)\left(g_2^d - 1\right)\right] - \frac{4\left(1 - d\right)}{d}\times\left[\frac{-d}{2\sqrt{\pi}\;L^d}\int_0^{\infty} dt\; t^{\frac{3}{2}}\sum_{n_0=1}^{\infty}\exp\left(-\frac{\beta^2}{4t}n_0^2\right) g_2' \;\, g_2^{d-1}\right]. \tag{2.2.4.14}$$

The divergence that occurs for d→3 in γ_2 is summarized by considering the divergence which arises at the normalization, this means:

$$\gamma_2 = -11\left[\frac{2}{(4\pi)^2(3-d)} + \frac{1}{(4\pi)^2}\int_0^{\bar t} dt\ t^{-1}\left(h_2^3 - 1\right)\right] + \frac{13}{3\sqrt{\pi}\ L^d}\times$$
$$\bar t^{\frac{+3}{2}} + \frac{4}{(4\pi)^2}\times \tag{2.2.4.15}$$
$$\int_0^{\bar t} dt\ t^{-2} h_2' h_2^2 - \frac{13}{2\sqrt{\pi}\ L^d}\int_{\bar t}^{\infty} dt\ t^{\frac{1}{2}}\left(g_2^3 - 1\right) - \frac{2}{\sqrt{\pi}\ L^d}\int_{\bar t}^{\infty} dt\ t^{\frac{3}{2}} g_2' g_2^2 +$$
$$\frac{1}{48\pi^2} - \frac{11}{16\pi^2}\log\ \bar t - \frac{11}{16\pi^2}\log\ (4\pi)$$

The related to heat part $\gamma_2'(T \neq 0)$ reads:

$$\gamma_2'(T \neq 0) = -\frac{(d+23)}{2}\left[\frac{2}{(4\pi)^{\frac{d+1}{2}}}\int_0^{\bar t} dt\ t^{\frac{1-d}{2}} h h_2^d - \frac{1}{\sqrt{\pi}\ L^d}\int_0^{\bar t} dt\ t^{\frac{1}{2}} h\right] - \frac{4(1-d)}{d}\times$$
$$\left[-\frac{d}{2(4\pi)^{\frac{d+1}{2}}}\int_0^{\bar t} dt\ t^{-\frac{d}{2}+\frac{1}{2}}\ \mathrm{hh}_2^{d} - \frac{d}{(4\pi)^{\frac{d+1}{2}}}\int_0^{\bar t} dt\ t^{-\frac{d+1}{2}} h h_2' h_2^{d-1}\right]$$
$$-\frac{(d+23)}{2}\left[\frac{1}{\sqrt{\pi}\ L^d}\int_{\bar t}^{\infty} dt\ t^{\frac{1}{2}} h\left(g_2^d - 1\right)\right] - \frac{4(1-d)}{d}\left[-\frac{d}{2\sqrt{\pi}\ L^d}\int_{\bar t}^{\infty} dt\ \left(t^{\frac{3}{2}}\times\right.\right.$$
$$\left.\left. h g_2' g_2^{d-1}\right)\right].$$

(2.2.4.16)

When we put (B.7), (B.8) and (B. 9) in (2.2.4.4) we get the following expression of γ_3:

$$\gamma_3' = -\frac{3(d-1)}{8}\frac{1}{L^d\beta\Gamma(2)}\int_0^{\infty} dt\ t\, g_1\left(g_2^d - 1\right) + \frac{3(d-1)(-d)}{dL^d\beta\Gamma(3)}\times$$
$$\int_0^{\infty} dt\ t^2\, g_1 g_2' g_2^{d-1} - (d-1)\frac{9}{L^d\beta\Gamma(4)}\int_0^{\infty} dt\ t^3\, g_1 g_2' g_2^{d-1}.$$

(2.2.4.17)

We put, after that (A. 12) in (2.2.4.17). γ_3 , at that time, is divided into two parts:

$$\gamma_3' = \gamma_3 + \gamma_3'\ (T \neq 0) \tag{2.2.4.18}$$

By this, it is:

$$\gamma_3 = -\frac{3}{16\pi^2}\int_0^{\bar t} dt\ t^{-3} h_2'^2 h_2 - \frac{3}{8\sqrt{\pi}\ L^3}\int_{\bar t}^{\infty} dt\ t^{\frac{1}{2}}\left(g_2^3 - 1\right) - \frac{6}{\sqrt{\pi}\ L^3}\times$$
$$\int_{\bar t}^{\infty} dt\ t^{\frac{3}{2}} g_2' g_2^2 - \frac{3}{2\sqrt{\pi}\ L^3}\int_{\bar t}^{\infty} dt\ t^{\frac{5}{2}} g_2'^2 g_2 + \frac{1}{4\sqrt{\pi}}\bar t^{\frac{3}{2}}$$

(2.2.4.19)

and:

$$\gamma_3'\ (T \neq 0) = -\frac{(d-1)}{2}\left[\frac{2}{(4\pi)^{\frac{d+1}{2}}}\int_0^{\bar t} dt\ t^{\frac{1-d}{2}} h h_2^d - \frac{1}{\sqrt{\pi}\ L^d}\int_0^{\bar t} dt\ t^{\frac{1}{2}} h\right]$$
$$+\frac{3(d-1)}{d}\left[\frac{-3}{2(4\pi)^{\frac{d+1}{2}}}\int_0^{\bar t} dt\ t^{-\frac{d}{2}+\frac{1}{2}}\ \mathrm{hh}_2^{\mathrm{d}} - \frac{3}{(4\pi)^{\frac{d+1}{2}}}\int_0^{\bar t} dt\ t^{-\frac{d+1}{2}} h h_2' h_2^{d-1}\right]$$
$$-\frac{3(d-1)}{8}\left[\frac{1}{\sqrt{\pi}\, L^d}\int_{\bar t}^{\infty} dt\ t^{\frac{1}{2}} h\left(g_2^d - 1\right)\right] - \frac{(d-1)}{d}\left[\frac{3}{\sqrt{\pi}\ L^d}\times\int_{\bar t}^{\infty} dt\ t^{\frac{3}{2}} h g_2' g_2^{d-1}\right]$$

$$-\frac{3}{2}(d-1)\left[\frac{1}{2(4\pi)^{\frac{d+1}{2}}}\int_0^{\bar{t}} dt\ t^{-\frac{d}{2}+\frac{1}{2}}\ \mathrm{hh}_2^{\mathrm{d}}+\frac{2}{(4\pi)^{\frac{d+1}{2}}}\times\int_0^{\bar{t}} dt\ t^{-\frac{d+1}{2}}\ hh_2' h_2^{d-1}\right]$$
$$-\frac{3(d-1)}{(4\pi)^{\frac{d+1}{2}}}\int_0^{\bar{t}} dt\ t^{-\frac{d+3}{2}}\ h h_2'^2\ h_2^{d-1}\ -\frac{3(d-1)}{2\sqrt{\pi}\ L^d}\int_{\bar{t}}^{\infty} dt\ t^{\frac{5}{2}}\ h g_2'^2\ g_2^{d-2}\ .$$
(2.2.4.20)

We similarly calculate γ_4 . First put (B.10) in (2.2.4.5), we get then:

$$\gamma_4' = \frac{(d-1)}{L^d\ \ \beta\Gamma(4)}\int_0^{\infty} dt\ t^3\ g_1\left(g_2''\ g_2{}^{d-1} - 3g_2'^2\ g_2^{d-2}\right). \tag{2.2.4.21}$$

Then, we put (A,12) in (2.2.4.21) , So , γ_4 is divided into two parts:

$$\gamma_4'\ = \gamma_4 + \gamma_4'\ (T \neq 0)\ . \tag{2.2.4.22}$$

It is , so:

$$\gamma_4 = -\frac{1}{48\,L^d}\int_0^{\bar{t}} dt\ \ t^{-\frac{d+3}{2}}\ h_2^{d-2}\left(h_2''\ \ h_2 - 3h_2'^2\right)$$
$$-\frac{1}{6L^d\sqrt{\pi}}\int_{\bar{t}}^{\infty} dt\ \ t^{\frac{5}{2}} g_2^{d-2}\left(g_2'' g_2 - 3g_2'^2\right) \tag{2.2.4.23}$$

$$\gamma_4'(T\ \neq\ 0) = -\frac{1}{32\ L^d}\int_0^{\bar{t}} dt\ \ t^{-\frac{d+3}{2}}\ h_2^{d-2}\left(h_2''\ \ h_2\ - 3h_2'^2\right)$$
$$-\frac{1}{6\sqrt{\pi}\ \ L^d}\int_{\bar{t}}^{\infty} dt\ \ t^{\frac{5}{2}} hg\ _2^{d-2}\left(g_2'' g_2\ - 3\,g_2'^2\right). \tag{2.2.4.24}$$

RESULTS AND DISSCUUSSION

The minimum of the classical potential is acceptable when the eight fields of gauge B_i^a are parallel in the eight degree of freedom (SU (3)- indices). This is what one calls toron-valley. We make this valley parameter throughout the length B_i of these eight parallel gauge fields.

The effective potential of toron is devoted to the homogenous gauge fields through this combination.

$$B_i^a = B_i\ \ n^a \tag{2.3.1}$$

That is $n^a . n^a = 1$.

The coefficients $\gamma_1'\ ,\gamma_2'\ \ ,\gamma_3'\ ,\gamma_4'$ are numerically calculated for different values of temperature. Meanwhile, the coefficients $\gamma_1 ,\gamma_2\ ,\gamma_3\ ,\gamma_4$ are independent of torus-length L. In order to calculate $\gamma_1'\ ,\gamma_2'\ \ ,\gamma_3'\ ,\gamma_4'$ we take L= 1. When calculating γ_2' and γ_3' , one can prove that the integrations for $\beta \geq 0.1$ are very small. So, we need to take the integrations only in the range $0 \leq t \leq 1$. The numeral results of the coefficients $\gamma_1'\ ,\gamma_2'\ \ ,\gamma_3'\ ,\gamma_4'$ are given in table (1). One can see that $\gamma_1'\ ,\gamma_2'\ \ ,\gamma_4'$ are degreased by increasing the temperature, while γ_3' is increased by the increasing of temperature. We have the effective potential of Toron:

$$V^{Tor}_{eff(1)}(B_1) = \gamma'_1 (B_1)^2 + \gamma'_3 S^{abcd} \delta^{ab} \delta^{cd} (B_1)^4 + \gamma'_4 S^{abcd} \delta^{ab} \delta^{cd} (B_1)^4$$

$$V^{Tor}_{eff(1)}(B_1) = \gamma'_1 (B_1)^2 + 60(\gamma'_3 + \gamma'_4)\ (B_1)^4 \quad ; S^{abcd} \delta^{ab} \delta^{cd} = 60 \qquad (2.3.2)$$

It is drawn in figure (1). The drawn potential of Toron is sloping with temperature. This means that the valley becomes deeper with the increasing of the temperature. In order to be able, discuss the behavior of the gauge theory, we have to know the behavior of the effective potential or the behavior of the gauge fields with temperature. For that, we consider the second derivative of the effective potential:

$$\frac{\partial^2 v_{eff(1)}}{\partial B^3_2 \partial B^3_2} = 2\gamma'_1 \left(\frac{1}{g^2(L)} + \gamma'_2 \right) \Big[(f^{123})^2 B^1_i B^1_i + (f^{345})^2 B^3_i B^3_i + (f^{367})^2 B^3_i B^3_i +$$
$$f^{123})^2 B^2_j B^2_j + (f^{345})^2 B^4_j B^4_j + (f^{367})^2 B^6_j B^6_j \Big] + 2\gamma'_3 \Big[s^{ab33} B^a_i B^b_i + s^{33cd} B^c_j B^d_j +$$
$$s^{3b3d} B^b_2 B^d_2 + s^{3bc3} B^b_2 B^c_2 + s^{a33d} B^a_2 B^d_2 + s^{a3c3} B^a_2 B^c_2 \Big] + 4\gamma'_4 \Big[s^{ab33} B^a_2 B^b_2 + s^{33cd} B^c_2 B^d_2 +$$
$$s^{a3c3} B^a_2 B^b_2 + s^{a33d} B^a_2 B^d_2 + s^{3bc3} B^b_2 B^c_2 + s^{3b3d} B^b_2 B^d_2 \Big]$$

(2.3.3)

From that, we draw:

$$\left. \frac{\partial^2 v_{eff\,(1)} \left(B^2_1 \right)}{\partial\ B^3_2 \partial\ B^3_2} \right|_{B^3_2 = 0} = 2\gamma'_1 + \gamma'_2 \left(B^2_1 \right)^2 + 3\gamma'_3 \left(B^2_1 \right)^2 \qquad (2.3.4)$$

in figure (2). Meanwhile:

$$g^2(L) = \frac{-1}{2b_0 \log(\Lambda_{ms}\ L\ )} - \frac{b_1 \log[-2\log(\Lambda_{ms}\ L\ )]}{4b^3_0 [\log(\Lambda_{ms}\ L\ )]^2} + \text{———} \qquad (2.3.5)$$

is the coupling constant which is defined throughout the minimum subtraction of dimension – normalization [23]. Constants b_0, b_1 have the following values:

$$b_0 = \frac{22}{3}(4\pi)^2 \quad , b_1 = \frac{136}{3}(4\pi)^4 \ . \qquad (2.3.6)$$

Figure (2) shows that the bend is decreasing by the increasing of temperature. For the low temperature, the valley from the inside is narrower than it is from the outside. This is confirmed till about $Z = \dfrac{L}{\beta_c} = 2.4$. (2.3.7)

$$\beta_C = \frac{L}{2.4} = \frac{1}{2.4} = 0.4166666667\,f = 2.1116666667\,Gev^{-1}$$

The critical temperature $T_C = \dfrac{1}{\beta_c} = 0.4735595896$ Gev $= 5.6827150752 \times 10^{12}$ K

This result identified the result in [17.33].

For 2.4 < Z ; the inside of the valley becomes wider than its outside. Qualitatively, the change in the valley-configuration indicates the phase-transition which was investigated in [31-33]. The coefficient γ'_2 in table (1) also indicates this phase-transition.

Appendix A: The heat kernels: First, we will define the heat kernels:

$$g_1(t)=\sum_{n_0=-\infty}^{\infty}\exp\left[-t\left(\frac{2\pi}{\beta}\right)^2 n_0^2\right] \quad \text{(A.1)}$$

$$g_2(t)=\sum_{n=-\infty}^{\infty}\exp\left[-t\left(\frac{2\pi}{L}\right)^2 n^2\right] \quad \text{(A.2)}$$

$$g_3(t,B_i)=\sum_{n=-\infty}^{\infty}\exp\left[-t\left(\frac{2\pi}{L}n+B_i\right)^2\right] \quad \text{(A.3)}$$

$$\mathbf{g_3(t,0)=g_3(t)=g_2(t)} \quad \text{(A.4)}$$

One can derive the properties of g_1 , g_2 and g_3 for t is small by the helping of Possion-resummation:

$$\sum_{n=-\infty}^{\infty}\exp\left(-\pi n^2 A+2n\pi AS\right)=\frac{1}{\sqrt{A}}\exp\left(\pi A s^2\right)\sum_{n=-\infty}^{\infty}\exp\left(-\pi A^{-1}n^2-2i\pi ns\right).$$

(A.5)

We easily find of that:

$$g_1(t)=\frac{\beta}{\sqrt{4\pi t}}\sum_{n_0=-\infty}^{\infty}\exp\left(-\frac{\beta^2}{4t}n_0^2\right) \quad \text{(A.6)}$$

$$g_2(t)=\frac{L}{\sqrt{4\pi t}}\sum_{n=-\infty}^{\infty}\exp\left(-\frac{L^2}{4t}n^2\right)$$

(A.7)

$$g_3(t,B_i)=\frac{L}{\sqrt{\pi t}}\sum_{n=-\infty}^{\infty}\cos(nB_iL)\exp\left[\left(-\frac{L^2}{4t}n^2\right)\right]+\frac{1}{\sqrt{4\pi t}}.$$

(A.8)

From (A,8) for the heat kernel g_3 , we get these following relations:

$$g_3(t,-B_i)=g_3(t,B_i)$$

$$g_3(t,B_i+2\pi)=g_3(t,B_i).$$

This concludes to:

$$\mathbf{g_3(t,B_i)=\sum_{n=0}^{\infty}C_n(t)\cos(nB_i)}$$

(A.9)

The $C_n(t)$ can be stated from (A,8):

$$C_0 = \frac{1}{\sqrt{4\pi t}}$$

$$C_n(t) = \frac{L}{\sqrt{\pi t}} \exp\left(- \frac{L^2 n^2}{4t} \right) \; ; \; n \geq 1 \; . \tag{A.11}$$

One can , by the helping of $h_1(u)$ and $h_2(u)$, write g_1 and g_2:

$$g_1(t) = \frac{\beta}{\sqrt{4\pi t}} h_1(u) \tag{A.12}$$

$$g_2(t) = \frac{\beta}{\sqrt{4\pi t}} h_2(u) \tag{A.13}$$

Meanwhile, $u, h_1(u)$ and $h_2(u)$ are defined like this:

$$u = \frac{1}{t} \tag{A.14}$$

$$h_1(u) = 1 + 2h(u)$$

$$h_2(u) = \sum_{n=-\infty}^{\infty} \exp\left(- \frac{L^2}{4} n^2 \right) u \; . \tag{A.15 }$$

h has the following form:

$$h\ (u) = \sum_{n_0=1}^{\infty} \exp\left(- \frac{\beta^2}{4} n_0^2 \right) u \; . \tag{A.16}$$

At $t \to 0$, one can use (A.6) and (A.7) which are written like this

$$g_1 = \frac{\beta}{\sqrt{4\pi\, t}} \left[1 + 0\left(\exp\left(- \frac{\beta^2}{4t} \right) \right) \right] \tag{A.17}$$

$$g_2 = \frac{L}{\sqrt{4\pi\, t}} \left[1 + 0\ \left(\exp\ \left(- \frac{L^2}{4t} \right) \right) \right] . \tag{A.18}$$

But, for $t \to \infty$ one can use (A.1) and (A.2) which are written like the following:

$$g_1 = 1 + 0\left[\exp\left(-t\left(\frac{2\pi}{\beta} \right)^2 \right) \right] \tag{A.19}$$

$$g_2 = 1 + 0\left[\exp\left(-t\left(\frac{2\pi}{L} \right)^2 \right) \right] \tag{A.20}$$

Now, we will calculate the derivatives of g_2 to $t \to 0$:

$$g_2' = -\frac{L}{2\sqrt{4\pi t}} t^{-\frac{3}{2}} \quad h_2(u) - \frac{L}{\sqrt{4\pi t}} t^{-\frac{5}{2}} \quad h_2'(u) \tag{A.21}$$

$$g_2'^2 = \frac{L^2}{4(4\pi)} t^{-3} \; h_2^2(u) + \frac{L^2}{(4\pi)} t^{-4} \; h_2 h_2' + \frac{L^2}{(4\pi)} t^{-5} \; h_2'^2(u)$$

(A.22)

$$g_2'' = \frac{3}{4} \; \frac{L}{4\sqrt{4\pi}} t^{-\frac{5}{2}} \; h_2^2(u) + \frac{3L}{\sqrt{4\pi}} t^{-\frac{7}{2}} \; h_2'(u) + \frac{L}{\sqrt{4\pi}} t^{-\frac{9}{2}} \; h_2''(u)$$

(A.23)

$$\left(g_2'' g_2 - 3g_2'^2\right) = \left[\frac{3}{4}\frac{L}{\sqrt{4\pi}} t^{-\frac{5}{2}} \; h_2^2(u) + \frac{3L}{\sqrt{4\pi}} t^{-\frac{7}{2}} \; h_2'(u) + \frac{L}{\sqrt{4\pi}} t^{-\frac{9}{2}} \times h_2''(u)\right] \times$$

$$\left(\frac{L}{\sqrt{4\pi}} t^{-\frac{1}{2}} \; h_2(u)\right) - 3\left[\frac{L^2}{4(4\pi)} t^{-3} \; h_2^2(u) + \frac{L^2}{(4\pi)} t^{-4} \; h_2 h_2' + \frac{L^2}{(4\pi)} t^{-5} \; h_2'^2(u)\right]$$

$$= \frac{L^2}{(4\pi)} t^{-5} \left[h_2''(u) h_2(u) - 3h_2'^2(u)\right].$$

(A.24)

Appendix B: calculation of sums of the discrete momentums on the torus, one can write for Bosons:

$$\left[\; K_i = \frac{2\pi}{L} n_i \;\right]$$

$$K_0 = \frac{2\pi}{\beta} n_0$$

$$\sum_{k_0}\sum_{\vec{k}} = \left(\frac{2\pi}{\beta}\right)\left(\frac{2\pi}{L}\right)^d \sum_{n_0}\sum_{\vec{n}} \tag{B.1}$$

this concludes to:

$$\frac{1}{(2\pi)^{d+1}} \sum_{k_0}\sum_{\vec{k}} \frac{1}{\left(k_0^2 + \left|\vec{k}\right|^2\right)^{\frac{s}{2}}} = \frac{1}{\beta L^d} \sum_{n_0}\sum_{\vec{n}} \frac{1}{\left(\left(\frac{2\pi}{\beta}\right)^2 n_0^2 + \left(\frac{2\pi}{L}\right)^2 n^2\right)^{\frac{s}{2}}} .$$

(B, 2)

Now, we will rewrite these coefficients as integration on the heat kernels. First, we calculate the following integration:

$$\int_0^\infty dt \; t^{\frac{s}{2}-1} g_1 = \sum_{n_0=-\infty}^{\infty} \int_0^\infty dt \; t^{\frac{s}{2}-1} \exp\left(-t\left(\frac{2\pi}{\beta}\right)^2 n_0^2\right) \tag{B.3}$$

$$\int_0^\infty dt\ t^{\frac{s}{2}-1} g_1 = \int_0^\infty dt\ t^{\frac{s}{2}-1} \exp(-t) \sum_{n_0=-\infty}^{\infty} \left(\left(\frac{2\pi}{\beta} \right)^2 n_0^2 \right)^{-\frac{s}{2}} . \tag{B.4}$$

This concludes to:

$$\sum_{n_0=-\infty}^{\infty} \frac{1}{\left(\left(\frac{2\pi}{\beta} \right)^2 n_0^2 \right)^{\frac{s}{2}}} = \frac{1}{\Gamma\left(\frac{s}{2} \right)} \int_0^\infty dt\ t^{\frac{s}{2}-1} g_1 \tag{B.5}$$

Meanwhile, it is:

$$\Gamma\left(\frac{s}{2} \right) = \int_0^\infty dt\ t^{\frac{s}{2}-1} \exp(-t) \tag{B.6}$$

(B.3), (B.4) and (B.5) conclude to:

$$\frac{1}{(2\pi)^{d+1}} \sum_{k_0} \sum_{\vec{k}=\vec{0}} \frac{1}{\left(k_0^2 + \left| \vec{k} \right|^2 \right)^{\frac{s}{2}}} = \frac{1}{\beta L^d} \frac{1}{\Gamma\left(\frac{s}{2} \right)} \int_0^\infty dt\ t^{\frac{s}{2}-1} g_1(t) g_2^d(t) . \tag{B.7}$$

After that, one easily finds that these following relations are really active:

$$\frac{1}{(2\pi)^{d+1}} \sum_{k_0} \sum_{\vec{k}\neq\vec{0}} \frac{1}{\left(k_0^2 + \left| \vec{k} \right|^2 \right)^{\frac{s}{2}}} = \frac{1}{\beta L^d} \frac{1}{\Gamma\left(\frac{s}{2} \right)} \int_0^\infty dt\ t^{\frac{s}{2}-1} g_1 (g_2^d - 1) \tag{B.8}$$

$$\frac{1}{(2\pi)^{d+1}} \sum_{k_0} \sum_{\vec{k}\neq\vec{0}} \frac{k_1^2}{\left(k_0^2 + \left| \vec{k} \right|^2 \right)^{\frac{s}{2}}} = \frac{-1}{\beta L^d} \frac{1}{\Gamma\left(\frac{s}{2} \right)} \int_0^\infty dt\ t^{\frac{s}{2}-1} g_1 g'_2 g_2^{d-1} \tag{B.9}$$

$$\frac{1}{(2\pi)^{d+1}} \sum_{k_0} \sum_{\vec{k}\neq\vec{0}} \frac{k_1^2 k_2^2}{\left(k_0^2 + \left| \vec{k} \right|^2 \right)^{\frac{s}{2}}} = \frac{1}{\beta L^d} \frac{1}{\Gamma\left(\frac{s}{2} \right)} \int_0^\infty dt\ t^{\frac{s}{2}-1} g_1 g_2'^2 g_2^{d-2} \tag{B.10}$$

$$\frac{1}{(2\pi)^{d+1}} \sum_{k_0} \sum_{\vec{k}\neq\vec{0}} \frac{k_1^4}{\left(k_0^2 + \left| \vec{k} \right|^2 \right)^{\frac{s}{2}}} = \frac{1}{\beta L^d} \frac{1}{\Gamma\left(\frac{s}{2} \right)} \int_0^\infty dt\ t^{\frac{s}{2}-1} g_1 g_2'^2 g_2^{d-2} \tag{B.11}$$

$$\frac{1}{(2\pi)^{d+1}}\sum_{k_0}\sum_{\vec{k}\neq\vec{0}}\frac{k_1^4-3k_1^2k_2^2}{\left(k_0^2+\left|\vec{k}\right|^2\right)^{\frac{s}{2}}}=\frac{1}{\beta L^d}\frac{1}{\Gamma\left(\frac{s}{2}\right)}\int_0^\infty dt\ t^{\frac{s}{2}-1}g_1\left(g_2{}''g_2^{d-1}\right.$$

$$\left.-3g_2'^2g_2^{d-2}\right).$$

(B.12)

Appendix C: Group theories relations: Lie – Algebra $SU(3)$ consists of all complex 3×3 matrixes x with:

$$x^+=-x\ ,Tr\ (x)=0\ .\tag{C.1}$$

The base, for such matrixes, is T^a ; $a=1,2,3,.........,8$.

$$T^a=\frac{\lambda^a}{2}\ ,\tag{C.2}$$

these λ^a are the Gell – Mann- matrixes:

$$\lambda^1=\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix},\lambda^2=\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix},\lambda^3=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix},\lambda^4=\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix},\tag{C.3}$$

$$\lambda^5=\begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix},\lambda^6=\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix},\lambda^7=\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix},\lambda^8=\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$$

The matrixes T^a fulfill:

$$Tr\ \left(T^aT^b\right)=-\frac{1}{2}\delta_{ab}\ .\tag{C.4}$$

The structure constant is defined throughout:

$$\left[T^a,T^b\right]=if^{abC}T^C\tag{C.5}$$

When x is an element of Lie – Algebra $SU(3)$, it is after that:

$$x=x^aT^a\ .\tag{C.6}$$

In the adjoint representation, it is:

$$(adx)^{ab}=if^{acb}\ x^C\ .\tag{C.7}$$

In the following notes, we will point some of the adjoint representation rules:

$$[adx,ady]=ad[x,y]\tag{C.8}$$

$$Tr(adx\ \ ady)=-6Tr(xy)\tag{C.9}$$

$$Tr\ (adB_i\ \ adB_i)=3B_i^aB_i^a\tag{C.10}$$

$$Tr\left(adB_i adB_i adB_j adB_j\right) = S^{abcd} B_i^a B_i^b B_j^C B_j^d + \frac{1}{2} F_{ij}^a(B) F_{ij}^a(B) \quad (C.11)$$

$$Tr\left(\left[adB_i, adB_j\right]\left[adB_i, adB_j\right]\right) = -3F_{ij}^a(B) F_{ij}^a(B) \quad (C.12)$$

$$Tr\left(\left(adF_{ij}(B)\right)^2\right) = 3\left(F_{ij}^a(B)\right)^2 \quad (C.13)$$

$$Tr\left(adB_i adB_j adB_k adB_\ell\right) = S^{abcd} B_i^a B_j^b B_k^c B_\ell^d \quad (C.14)$$

Meanwhile, it is:

$$S^{abcd} = \frac{3}{12}\left(d^{abe} d^{cde} + d^{ace} d^{bde} + d^{ade} d^{bce}\right) + \frac{2}{3}\left(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}\right) \quad (C.15)$$

$$F_{ij}(B) = i\left[B_i, B_j\right] \quad (C.16)$$

and:

$$adF_{ij}(B) = i\left[adB_i, adB_j\right] \quad (C.17)$$

$$F_{ij}^a(B) = f^{abc} B_i^b B_j^c \; . \quad (C.18)$$

Note: some of the relations are only applied when B_i is constant.

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