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ROBUST CONTROL OF AIRCRAFTS USING LINEAR MATRIX INEQUALITIES

***Victor A F de Campos, Alain S Potts and Luiz R T da Silva**

Universidade Federal do ABC (UFABC)

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Corresponding author:
Victor A F de Campos,

ABSTRACT

This article presents a new method for robust control of aircraft dynamic models using linear matrix inequalities and pole placement. The models are written in the form of polytopes, that describes the dynamic system linearized around some operating point. By using ljnear matrix inequalities, different control requisites, such as pole placement at different regions of the complex plane and norm minimization, can be grouped in one structure. Results of application for helicopter and fighter aircraft models show that the methodology guarantees stability and performance robustness for both dynamic systems.

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INTRODUCTION

Dynamic performance and robust control of dynamical systems are subjects of great importance in Engineering. Techniques like LQG / LTR (Loop Transfer Recovery), μ synthesis and Linear Quadratic Regulator were explored in many past researches. Nevertheless, the most flexible technique in terms of grouping different requisites involves the use of Linear Matrix Inequalities (LMIs). LMIs have been used in many control applications (Mackenroth, 2004). (Chilali, Gahinet and Apkarian, 1999) and (Chilali and Gahinet, 1996) apply LMIs to robust pole placement in generic systems, but the controllers generated are full order, and the formulation does not permit to define the structure of the controller. In (de Campos, Cruz and Zanetta, 2012), robust pole placement is done together with minimization of the norm of controllers static gain matrix, with fixed order controllers. Aircrafts are dynamic systems. Generally, the flight control systems are designed using mathematical models, which are linearized around various operating points. The controller parameters are pre-programmed or varied in terms of the flight conditions. In the area of robust control applied to helicopters, there are a predominance of H2 and H∞ controllers. Two works that use H∞ method to the helicopters control are (Yoneyama and Kikuchi, 2002) and (Wang, Lu and Zhong, 2013). Another technique applied to control of helicopters is the Linear Quadratic Regulator (LQR). In (Shen *et al*, 2009), the LQR is applied to the design of a controller which stabilizes the 3 Euler angles of a helicopter. A robust – adaptive controller based on neural networks was proposed on (Razzaghian, 2018).

In the area of robust control applied to fighter aircrafts, (Holhjem, 2012) develops an adaptive control technique for the longitudinal model of a F16 aircraft, linearized around an operating point. The longitudinal model of an aircraft was also used by (Xu *et al*, 2014) to design an adaptive-robust controller. Various robust techniques have been used for aircrafts controllers design, for example the linear quadratic regulator (LQR), the H∞ controller and the μ synthesis (structured singular value). In (Fravolini at al, 2015), a linear control / adaptive control mixed strategy is applied to the longitudinal model of a F16 aircraft, yielding good results of robustness and performance. A controller design methodology that uses multi-objective optimization and LMIs for trajectory planning of aircrafts is developed in (Liao *et al*, 2002). Despite the different types of control strategies applied to aircrafts, the most flexible technique in terms of guaranteeing robustness and, at the same time, grouping different control strategies is the robust control through LMIs. In the technique that is proposed, we use polytopic models to assure robustness; besides, we can allocate the closed loop system eigenvalues in a specified region of the complex plane, what will guarantee good performance to the system at various operating points. Moreover, this technique makes possible to choose a priori the controller structure, an important feature of this work. In this field, (de Campos, Cruz and Zanetta, 2014), (Trajano da Silva, de Campos and Potts, 2020) and (Andrade, de Campos, Potts and Garcia, 2017) applied LMIs to the control of aircrafts, considering fixed controller structure. The robust control algorithm that will be presented in this work is more complete than this 3 preceding works, adding more robust features to the problem structure.

Mathematical Model of the System and Closed Loop System: The dynamic system model to be described here is written in the state space form. The dynamic model is the following:

$$\dot{x} = A.x + B.u$$
$$y = C.x \quad (1)$$

where x is the state vector, y is the output vector and u is the input vector. The i-th controller to be used has a pre defined structure:

$$K_i(s) = \frac{a_i.s^2+b_i.s+c_i}{(s+p_1)(s+p_2)\dots(s+p_n)} \quad (2)$$

where the poles $p1, \dots, pn$ are pre-determined. Another structures for the controllers are also possible. In this scheme, we have to obtain the gain and the zeros (given by the values of a_i, b_i and c_i) of the controller.

Our control policy consists on applying an output feedback to the system. K(s) is the controllers transfer function matrix, and G(s) is the transfer function matrix of the nominal system. The controllers matrix K(s) can be rewritten in state space form as:

$$\dot{x}_C = A_C.x_C + B_C.y$$
$$u = C_C.x_C + D_C.y \quad (3)$$

Matrices A_C and C_C are pre defined, once the poles of the controllers are also pre defined. Matrices B_C and D_C are the variables of the control problem. Applying the controller (3) to the system described by (1), we have the closed-loop dynamical system.

In order to simplify the formulation, we apply a transformation that turns the dynamic controller adjustment problem into a static controller problem. This method is the same presented in (Scavoni *et al*, 2001). We define the following matrices:

$$A_m = \begin{bmatrix} A & B.C_C \\ 0 & A_C \end{bmatrix} \quad B_m = \begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix} \quad C_m = \begin{bmatrix} C & 0 \end{bmatrix} \quad (4)$$

The static controller gain matrix is: $K_C = \begin{bmatrix} D_C \\ B_C \end{bmatrix}$ (5)

The modified system, which is equivalent to the closed loop system, is:

$$\dot{x}_m = A_m x_m + B_m u_m$$
$$y = C_m x_m \quad (6)$$

where $x_m = [x \ x_C]^T$ and the control law $u_m = K_C\, y$. The variable of the problem is the static output feedback gain matrix K_C.

Pole placement through linear matrix inequalities with pre-defined structure controllers: Linear Matrix Inequalities (or simply LMIs) are mathematical tools that have applications in robust control theory. For pole placement purposes, we apply the concept of LMI regions.

A real matrix A is D-stable, that is, has all of its eigenvalues in the LMI region D if and only if a real symmetric matrix Q exists such that (Boyd *et al*, 1994):

$$L \otimes Q + R \otimes (A.Q) + R^T \otimes (Q.A^T) < 0$$
$$Q > 0 \quad (7)$$

where L and R are matrices that define the LMI region.

LMI regions of interest in control applications are the conic sector, the semiplane and the region inside a semicircle, whose intersection can be seen in Figure 1.

By placing the dynamic system poles in the intersection of these 3 regions, we can guarantee that the closed loop system will have a minimum decay rate α, a minimum damping $\zeta = \cos\theta$, and a damped natural frequency of $\omega_d = r.\sin\theta$, besides of an undamped natural frequency of $\omega_n \le r$. That puts limits on the overshoot, the settling time and the rise time of the system. Therefore, to place the closed loop system poles in this region guarantees an adequate performance for the system.

Output feedback control and Pole Placement through LMI's

The general LMI for pole placement with fixed controller structure and output feedback is the following (de Campos, Cruz and Zanetta, 2012):

$$L \otimes Q + R \otimes (A_m Q + B_m N C_m) + R^T \otimes (Q A_m^T + C_m^T N^T B_m^T) < 0$$
$$Q > 0 \quad (8)$$

The variables are matrices Q and N. Once the LMI is solved, we can recover the controller (de Campos, Cruz and Zanetta, 2012):

$$M.C_m = C_m Q \quad (9)$$

$$K_C = N.M^{-1} \quad (10)$$

LMI (8) has to be solved for the 3 regions (defined by matrices L and R) shown in figure 1 simultaneously. Doing so, the poles of a dynamical system will be allocated in this specific region of the complex plane. Substituting the values of matrices L and R for the 3 regions of the complex plane, we have:

Conic sector with inner angle θ (de Campos, Cruz and Zanetta, 2012):

$$\begin{bmatrix} \sin\theta A_{cl} Q + \sin\theta.Q.A_{cl}^T & \cos\theta A_{cl} Q - \cos\theta.Q.A_{cl}^T \\ * & \sin\theta A_{cl} Q + \sin\theta.Q.A_{cl}^T \end{bmatrix} < 0 \quad (11)$$

where: $A_{cl} Q = A_m Q + B_m N C_m$, and * denotes symmetric term.

Disc of radius r:

$$\begin{bmatrix} -r.Q & A_m Q + B_m N C_m \\ * & -r.Q \end{bmatrix} < 0 \quad (12)$$

Semiplane Re(z) < α :

$$2\alpha.Q + A_m Q + B_m N C_m + Q.A_m^T + C_m^T N^T B_m^T < 0 \quad (13)$$

Summarizing the procedure for pre-defined structure controller design and pole placement, we have the following algorithm:

1. Firstly, evaluate matrices A_m, B_m and C_m of the modified system and define the performance specifications for the closed-loop system (that is, the values of θ, α and r). Solve the system of LMI's given by (11), (12), (13) and Q > 0 in the variables Q and N;
2. Compute the matrix M, using (9);
3. Compute the static gain matrix K_C using (10);
4. Recover the controller matrices D_C and B_C, considering (5);
5. Compute the transfer function of each controller, using A_C, B_C, C_C and D_C.

The Robust Algorithm

Dynamical systems like aircrafts are basically described by nonlinear models which are linearized around some operating points. The linearization is important, once the most powerful control techniques are applicable just for linear models. Having various operating points of the dynamical system, we have to guarantee that all of them are stable and present good performance. To assure it, we make use of polytopic models.

So, let the i-th system model linearized around an operating point be denoted by the triple (A_i , B , C). A polytope is the set Ω defined below (Boyd *et al*, 1994):

$$\Omega = \{A | A \in R^{nxn}, A = \sum_{i=1}^{m} \lambda_i A_i, \lambda_i \geq 0, \sum_{i=1}^{m} \lambda_i = 1\} \quad (14)$$

where n is the dimension of matrices Ai and m is the number of operating points. The matrices Ai are called vertices of the polytope. To ensure that the poles of any closed-loop system associated to a matrix $A \in \Omega$ will be in the region of the complex plane defined in figure 1, we have to solve m LMIs jointly in the same variables Q and N, that is:

$$min\|G(s)K(s)\|_2$$

Subject to:

$$\begin{bmatrix} sin\theta A_{cl,i}Q + sin\theta.Q.A_{cl,i}^T & cos\theta A_{cl,i}Q - cos\theta.Q.A_{cl,i}^T \\ * & sin\theta A_{cl,i}Q + sin\theta.Q.A_{cl,i}^T \end{bmatrix} < 0$$

$$\begin{bmatrix} -r.Q & A_{m,i}Q + B_m N C_m \\ * & -r.Q \end{bmatrix} < 0$$

$$2\alpha.Q + A_{m,i}Q + B_m N C_m + Q.A_{m,i}^T + C_m^T N^T B_m^T < 0$$

$$\begin{bmatrix} -k_N.I & N^T \\ N & -I \end{bmatrix} < 0$$

$$\begin{bmatrix} C_m Q C_m^T & I \\ I & k_Q.I \end{bmatrix} > 0$$

$$Q > 0$$

for i = 1, 2, 3, ..., m, with:

$$A_{m,i} = \begin{bmatrix} A_i & B.C_C \\ 0 & A_C \end{bmatrix}$$

$$A_{cl,i}Q = A_{m,i}Q + B_m N C_m \quad (15)$$

A_i , i = 1, 2, … , m are the state space matrices that define the mathematical model of the system, at various operating points. By solving the system of LMIs (15), it can be guaranteed that the closed-loop system poles will be in the region defined in Figure 1 for all the m operating points considered. At the same time, the minimization problem generates a feasible controller gain matrix, with minimum norm-2. Thus, this is the robust procedure for controllers design.

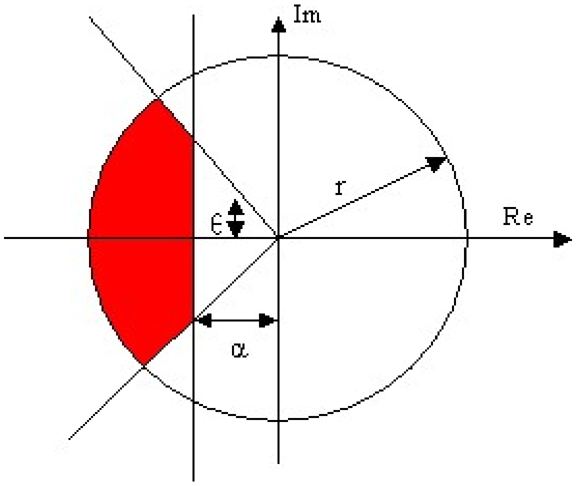

Figure 1. Intersection of the 3 regions of the complex plane for pole placement (half plane, semicircle and conic sector)

RESULTS

We present 2 applications of the robust procedure: the F16 aircraft and the helicopter Lynx.

F16 AIRCRAFT: The robust control algorithm was applied to a lateral dynamical model of a F16 aircraft, considering 3 operating points. These points are presented in Table 1. The linearized model of a F16 aircraft was obtained from (Stevens, Lewis and Johnson, 2015). The model has 7 states, 2 inputs and 4 outputs. The complete description of the model can also be found in (Andrade, de Campos, Potts and Garcia, 2017).

Table 1. Operating points for the F16 aircraft mathematical model

Operating points	
Condition	1
Nominal	2

We choose the following specifications for the robust control algorithm:

1. Controllers poles: - 2000
2. Damping factor: 0.5
3. Decay rate: 1
4. Radius: 2050

In Figure 2, we can observe the results of the robust controller proposed. The open loop system eigenvalues were plotted together with the closed loop system eigenvalues.

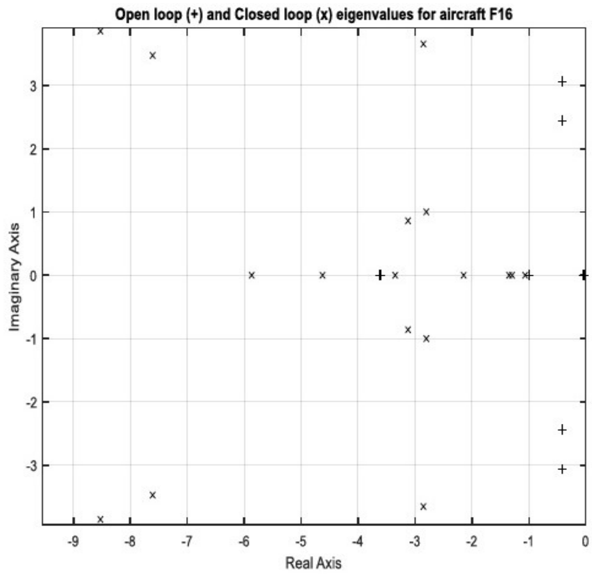

Figure 2. Eigenvalues of the open loop and closed loop systems (with robust controllers) – aircraft F16

The controller static gain matrix obtained is the following:

$$K_C = \begin{bmatrix} 0.4086000 & 0.0440000 & -1.270000 & 0.200000 \\ 1.6367000 & -0.0655000 & -4.190000 & -0.089000 \\ -0.000019 & 0.0000016 & -0.000199 & 0.000000 \\ 0.0384000 & -0.0032000 & 0.375500 & 0.001300 \\ 0.0002596 & -0.0000087 & -0.001000 & -0.000008 \\ -0.495600 & 0.0167000 & 1.950000 & 0.015400 \end{bmatrix}$$

The features of the closed loop system are (considering all the operating points):

Norm-2 (K_C) = 5.11

Minimum damping ratio = 0.61
Undamped natural frequency = 2000 rad/s
Decay rate = 1.07

The closed loop system response to initial conditions (all of them settled to the value of 50) can be seen in Figure 3. The settling time is near to 2.5 s for outputs 1, 2 and 3, and near to 4 s for output 4.

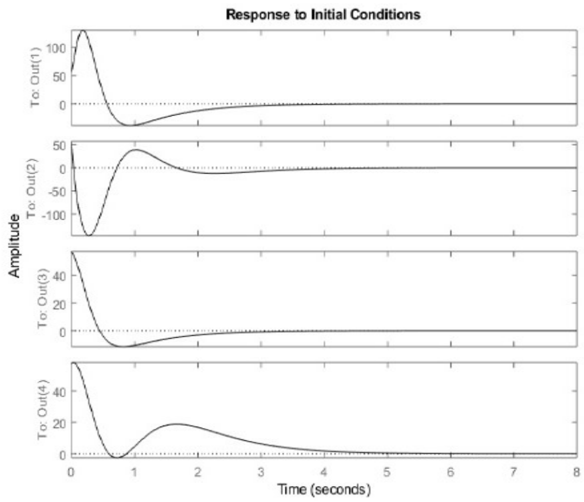

Figure 3. Response to initial conditions for the closed loop system model with robust controllers – aircraft F16

Helicopter Lynx: We use the state space linearized model of Westland Lynx MK7 helicopter, whose model is available at (Padfield, 2008). The helicopter linearized model has 8 state variables, 4 inputs and 4 outputs. The complete description of this model can also be found at (Trajano da Silva, de Campos and Potts, 2020). The original (nonlinear) equations were linearized around 3 operating points (Table 2). The mechanical equations, for a 6 degree of freedom helicopter, can be found at (Luo *et al*, 2003).

Table 2. Operating points for helicopter Lynx mathematical model

Operating Point	Flight Mode
1	Hover
2	Flight ahead 60 Knots
3	Flight ahead 100 Knots

We choose the following specifications for the robust control algorithm:

- Controllers poles: - 3000
- Damping factor (ξ): 0.1
- Decay rate (α): 0.02
- Radius (r): 3050

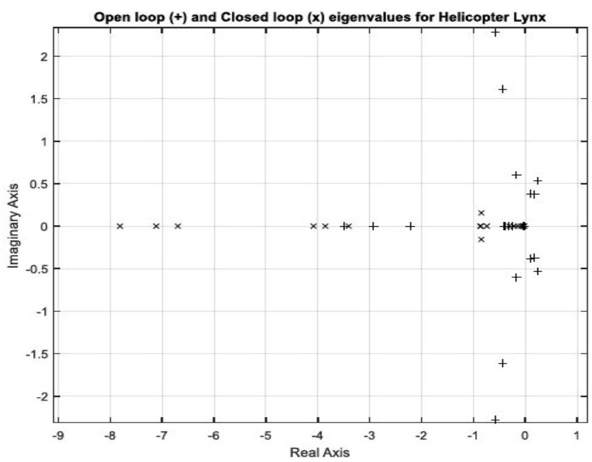

Figure 4. Eigenvalues of the open loop and closed loop systems (with robust controllers) – helicopter Lynx

In Figure 4, we can observe the results of the application of the robust controller proposed. The open loop system eigenvalues were plotted together with the closed loop system eigenvalues.

The controller static gain matrix obtained is the following:

$$K_C = \begin{bmatrix} 0.0154 & -0.1769 & 0.0049 & -0.0139 \\ -1.64 & -1.96 & 0.1271 & -0.0109 \\ 0.3073 & -0.1095 & 0.15 & -0.0339 \\ 12.11 & 12.53 & -1.46 & 6.3585 \\ 0.0293 & 0.0361 & 0.00076 & 0.00009 \\ -86.61 & -105.74 & -2.3 & -0.3 \\ 0.057 & 0.041 & 0.00014 & 0.00009 \\ -168.4 & -120.3 & -0.4723 & -0.2541 \\ 0.002 & 0.0013 & 0.00033 & 0 \\ -5.76 & -3.9172 & -0.9364 & 0.0328 \\ 0.00014 & 0.00017 & 0 & -0.000047 \\ -0.4296 & -0.51 & 0.0219 & 0.1396 \end{bmatrix}$$

The features of the closed loop system are (considering all operating points):

Norm-2 (K_C) = 246.92
Minimum damping ratio = 0.98
Undamped natural frequency = 3010 rad/s
Decay rate = 0.02

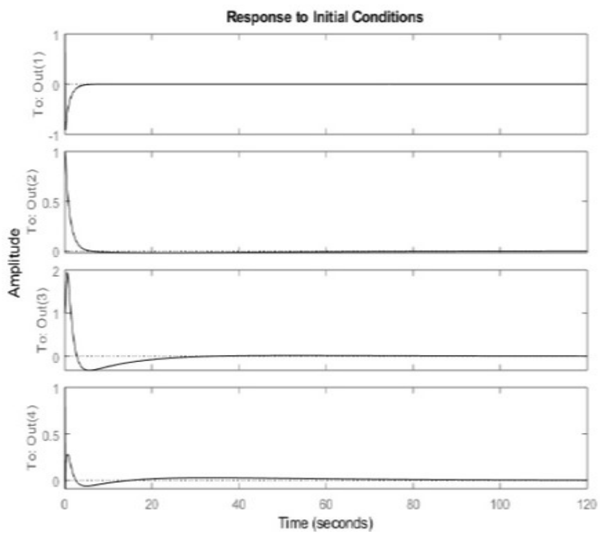

Figure 5. Response to initial conditions for the closed loop system with robust controllers – helicopter Lynx

The closed loop system response to initial conditions (all of them settled to the value of 1) can be seen in Figure 5. The settling time is near to 20 s.

DISCUSSION

The results presented show that the robust control strategies increased the damping and guaranteed stability for the aircrafts, at all operating points. In the case of the helicopter, it was instable, and the controllers turned the system into stability at all operating points. At the same time, the norm of the controllers gain matrix was bounded, what minimizes the control effort. Then, the robust controller increased the performance and guaranteed stability at all operating points, being also an optimal controller.

CONCLUSIONS

We proposed a new robust controller design method for aircrafts, which uses pole placement and LMIs, and whose results guarantee stability and performance for various operating points. We have shown 2 applications (fighter aircraft and helicopter), in both the robust stability and robust performance were guaranteed. At the same time, the norm-2 of the controller gain matrix was constrained, in order to avoid infeasible values for the controllers parameters. The controllers structure is totally free, and can be changed depending on the system demands. The results show that the performance of the aircrafts was greatly improved, and the robustness was also guaranteed. Some possibilities for further developments include the minimization of H infinity norm for the closed loop system, together with pole placement constraints, and the development of a nonlinear robust control strategy for the aircrafts.

REFERENCES

Andrade, J. P. P., de Campos, V. A. F., Potts, A. S. and Garcia, C., Damping improvement of a F16 aircraft through Linear Matrix Inequalities, *IFAC papers on line*, vol 50, pp. 3947 – 3952, 2017.

Boyd, S., El Ghaoui, L., Feron, E and Balakrishnan, V., Linear Matrix Inequalities in System and Control Theory, SIAM, 1994.

Chilali, M. and Gahinet, P. H∞ Design with Pole Placement Constraints: An LMI approach, IEEE Trans. Automatic Control, vol. 41, no. 3, pp. 358-367, Mar. 1996.

Chilali, M., Gahinet, P. and Apkarian, P. Robust Pole Placement in LMI Regions, IEEE Trans. Automatic Control, vol. 44, no. 12, pp. 2257-2269, Dec. 1999.

de Campos, V. A. F., Cruz, J. J. and Zanetta Jr., L. C., Pole Placement and Robust Adjustment of Power System Stabilizers through Linear Matrix Inequalities, Proceedings of the 2006 IEEE Power Systems Conference and Exposition, 2006.

de Campos, V. A. F., Cruz, J. J. and Zanetta Jr., L. C., Robust and optimal Adjustment of Power System Stabilizers through Linear Matrix Inequalities, *International Journal of Electrical Power and Energy Systems,* vol 42, pp. 478 – 486, 2012.

de Campos, V. A. F., Cruz, J. J. and Zanetta Jr., L. C., Robust control of dynamical systems using Linear Matrix Inequalities and norm bounded uncertainty, Journal of control, automation and electrical systems, vol 25, pp. 151 – 160, 2014.

Fravolini, M. L., Yucelen, T., Muse, J. and Valigi, P. Analysis and design of adaptive control systems with unmodeled input dynamics via multiobjective convex optimization, Proceedings of the 2015 American Control Conference (ACC), pp. 1579–1584, 2015.

Liao, F., Wang, J. L. and Yang, G. H. Reliable robust flight tracking control: an LMI approach, IEEE Trans Control Systems Technology, vol 10, no. 1, pp. 76– 89, 2002.

Luo, C. C., Liu, R. F., Yang, C. D. and Chang, Y. H., Helicopter H∞ control design with robust flying quality, *Aerospace Science and Technology*, vol 7, no. 2, pp. 159–169, 2003.

Mackenroth, U. Robust Control Systems, Springer, 2004.

Padfield, G. D., Helicopter flight dynamics: the theory and application of flying qualities and simulation modelling, John Wiley & Sons, 2008.

Razzaghian, A. Robust Adaptive Neural Network Control of Miniature Unmanned Helicopter, Proceedings of the 26th Iranian Conference on IEEE, 2018.

Scavoni, F. E., Silva, A. S., Trofino Neto, A. and Campagnolo, J. M. Design of Robust Power System Controllers using Linear Matrix Inequalities, Proceedings of the 2001 IEEE Porto Power Tech Conference, 2001.

Shen, N., Su, Z., Wang, X. and Li, Y., Robust controller design and hardware-in-loop simulation for a helicopter, Proceedings of the 4th IEEE Conference on Industrial Electronics and Applications, ICIEA, China, 2009.

Stevens, B. L., Lewis, F. L. and Johnson, E. N., Aircraft Control and Simulation: Dynamics, Controls Design, and Autonomous Systems, John Wiley & Sons, 2015.

Trajano da Silva, L. R., de Campos, V. A. F. and Potts, A. S. Robust Control for helicopters performance improvement: an LMI approach, *Journal of Aerospace Technology and Management,* vol 12, 2020.

Wang, X., Lu, G. and Zhong, Y. Robust H∞ attitude control of a laboratory helicopter, Robotics and Autonomous Systems, vol 61, no. 12, pp. 1247–1257, 2013.

Xu, B., Huang, X., Wang, D. and Sun, F. Dynamic surface control of constrained hypersonic flight models with parameter estimation and actuator compensation, *Asian Journal of Control*, vol 16, no. 1, pp. 162–174, 2014.

Yoneyama, J. and Kikuchi, Y. Robust control for uncertain systems with application to helicopter model, Proceedings of the 41st SICE Annual Conference, pp. 1187–1190, 2002.

****

Holhjem, Ø.H. L1 adaptive control of the inner control loops of an f-16 aircraft, Master Thesis, NTNU, Norge, 2012.