



## Full Length Research Article

### TREND ANALYSIS AND ESTIMATION OF HURST EXPONENT FOR AEROSOL TIME SERIES OF CHENNAI

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#### ABSTRACT

In the present paper, We use the rescaled range(R/S) analysis to estimate the Hurst exponent of PM10 data of Chennai region. The Hurst exponent, H, is a self-similarity parameter that measures the long-range dependence in a time series, and provides a measure of long-term nonlinearity. It is found that the behaviour of PM10 aerosol of Chennai is persistent as the value of Hurst Exponent for various regions in Chennai is found to be persistent. The trend analysis of PM10 using Mankendall showed trend in all the regions other than Adayar. region of Chennai.

#### INTRODUCTION

Among all the criteria air pollutants, particulate matter (SPM and RSPM) has emerged as the most critical pollutant in almost all urban areas of the country [www.cpcb.nic.in](http://www.cpcb.nic.in) Chennai is a coastal city. Chennai area 176 km sq, 2011 census 4.34 million, 2011 census 4.95million, vehicle population in 2007, 2.27• million. Weather is typically hot and humid. There is only a small variation between the seasons due to the location and proximity to the Indian Ocean Max. Temp.: 42° C and Min Temp: 20° C. These sources include large, medium and small-scale industries, household fuel use for cooking and heating, refuse burning, vehicular emissions, re-suspended road dust, construction activities, agricultural activity, naturally occurring dust and trans-boundary migration from other regions, etc. Diesel vehicles are known to be significant emitters of PM 10. In Chennai the contribution of PM 10 is due to is mainly due to transport, road dust and coal. With regard to PM 10, it was found that resuspension of unpaved and paved road dust contributed close to 68%, while vehicles contributed 12% to the pollution levels in Chennai. The time series consists of a set sequential numeric data taken at equally spaced intervals, usually over a period of time or space. The first step in time series analysis is to draw time series plot which provides a preliminary understanding of time behaviour of the series Marzuki Ismail *et al* (2011).

Time series analysis is a useful tool for better understanding of cause and effect relationship of environmental pollution. The main aim of time series analysis is to describe movement history of a particular variable in time. Many authors have tried to detect changing behaviour of air pollution through time using different techniques, Hies. T., Trewfffeisen *et al* (2003).

#### Data used

We have used PM10 AEROSOL data concentration of Chennai, Tamil Nadu for the period of 2013 to 2016 April.

#### MATERIALS AND METHODS

##### Hurst Exponent

The Hurst exponent is a parameter that quantifies the persistent or anti persistent (past trends to reverse in future) behaviour of a time series. It determines whether the given time series is completely random or has some long term memory.

##### Hurst Exponent

For calculating Hurst exponent, one must estimate the dependence of the rescaled range on the timespan  $n$  of observation. Various techniques have been adopted for calculating Hurst exponent. The oldest and best-known

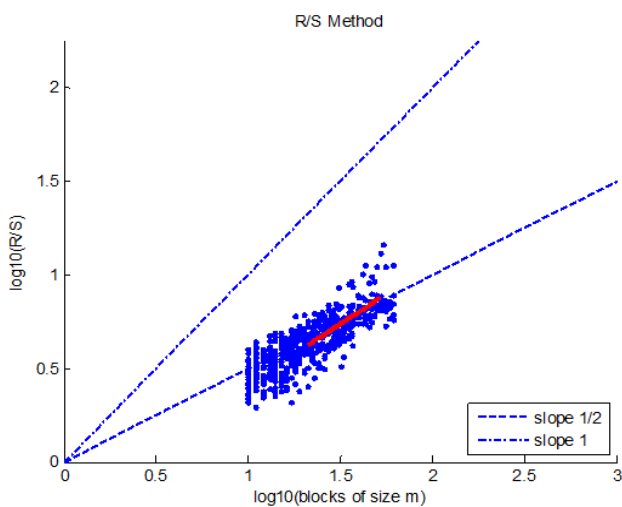
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method to estimate the Hurst exponent is R/S analysis. The rescaled analysis or R/S analysis is used due to its simplicity in implementation. It was proposed by Mandelbrot and Wallis (1969), based on the previous work of Hurst (1951): The R/S analysis is used merely because it has been the conventional technique used for geophysical time records Govindan *et al* (20004). A time series of full length  $N$  is divided into a number of shorter time series of length  $n = N, N/2, N/4 \dots$ . The average rescaled range is then calculated for each value of  $n$ . For a (partial) time series of length  $n$ , there scaled range is calculated as follows: Samuel Selvaraj *R et al* (2011)

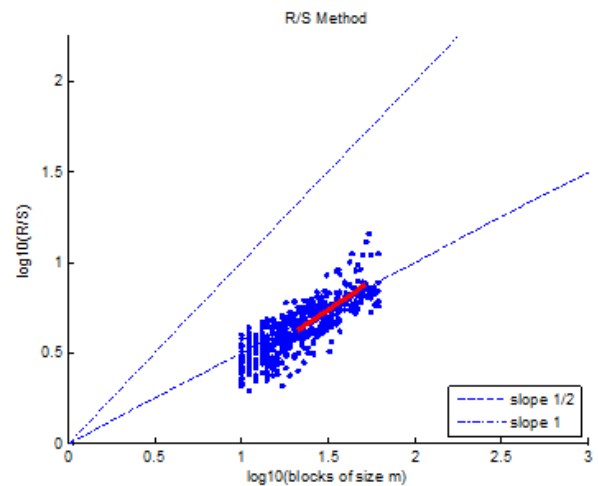
- Calculate the mean;
- Create a mean-adjusted series;
- Calculate the cumulative deviate series  $Z$ ;
- Compute the range  $R$ ;
- Compute the standard deviation  $S$ ;
- Calculate the rescaled range  $R(n)/S(n)$  and the average of over all partial time series of length  $n$ .

Hurst found that  $(R/S)$  scales by power-law as time increases, which indicates  $(R/S) n = c \cdot n^H$ , where  $c$  is a constant and  $H$  is called the Hurst exponent. To estimate the Hurst exponent, we plot  $(R/S)$  versus  $n$  in log-log axes. The slope of the regression line approximates the Hurst exponent. The values of the Hurst exponent range between 0 and 1. Based on the Hurst exponent value  $H$ , the following classifications of time series can be realized:  $H = 0.5$  indicates a random series;  $0 < H < 0.5$  indicates an anti-persistent series, which means an up value is more likely followed by a down value, and vice versa;  $0.5 < H < 1$  indicates a persistent series, which means the direction of the next value is more likely the same as current value (8) Alina, *et al* (2007)

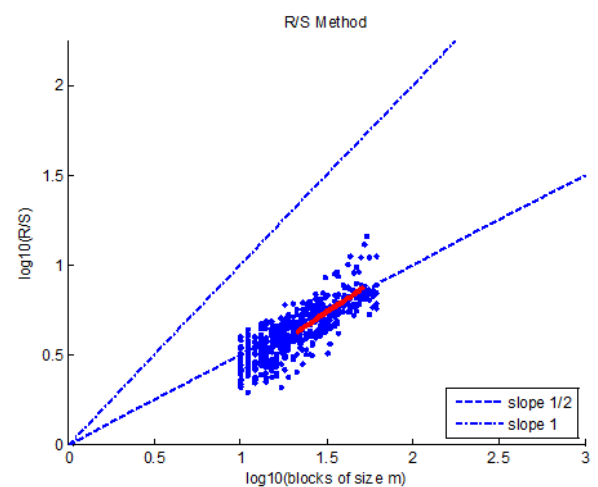
- If  $H=0.5$  then the series is completely random (Brownian motion)
- If  $0.5 < H \leq 1$  then the series is said to be persistent, i.e. if it was going up then down in the previous period, it will then more probably continue going up then down in the next period.
- If  $0 < H < 0.5$  then the series is said to be Anti-persistent. (up then down will more probably become down then up) Edgar E. Peters (1994)



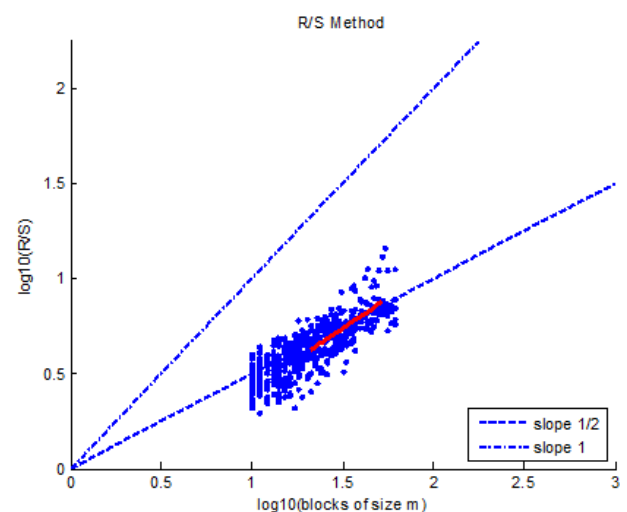
**Fig.1. Adyar**



**Fig. 2. Anna Nagar**



**Fig.3. T.Nagar**



**Fig.4. Kilpauk**

**RESULTS**

Figure 1 to 4 are the Hurst estimated slope by R/S method of Adyar, Anna Nagar, T.Nagar and Kilpauk regions of Chennai. As shown in above fig a graph is plotted for  $\log n$  vs  $\log R/S$

and the slope is calculated for the giventime series. The slope for the dataset is found to be .6471 for Adyar, 0.6693for Anna Nagar, 0.5844 for T.Nagar and 0.5768 for Kilpauk, and regions of Chennai respectively, which is the Hurst exponent. So, thePM10 aerosol ofChennai region follows persistent behaviour...

### Mann Kendall Trend Test

Mann Kendall test is a statistical test widely used for the analysis of trend in climatologicMavromatis T., (2011) and in hydrologic time series Yue S., Wang, C., 2004). There are two advantages of using this test. First, it is a non-parametric test and does not require the data to be normally distributed. Second, the test has low sensitivity to abrupt breaks due to inhomogeneous time series Tabari, H (2011). Any data reported as non-detects are included by assigning them a common value that is smaller than the smallest measured value in the data set (Blackwell Publishing 2012). According to this test, the null hypothesis H0 assumes that there is no trend (the data is independent and randomly ordered) and this is tested against the alternative hypothesis H1, which assumes that there is a trend (Onoz, 2012). The computational procedure for the Mann Kendall test considers the time series of n data points and  $T_i$  and  $T_j$  as two subsets of data where  $i = 1, 2, 3, \dots, n-1$  and  $j = i+1, i+2, i+3, \dots, n$ . The data values are evaluated as an ordered time series. Each data value is compared with all subsequent data values. If a data value from a later time period is higher than a data value from an earlier time period, the statistic  $S$  is incremented by 1. On the other hand, if the data value from a later time period is lower than a data value sampled earlier,  $S$  is decremented by 1. The net result of all such increments and decrements yields the final value of  $S$  [Drapela, 2011]. The Mann-Kendall S Statistic is computed as follows:

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^n \text{sign}(T_j - T_i)$$

$$\text{Sign}(T_j - T_i) = 1 \text{ if } T_j - T_i > 0$$

$$\text{Sign}(T_j - T_i) = 0 \text{ if } T_j - T_i = 0$$

$$\text{Sign}(T_j - T_i) = -1 \text{ if } T_j - T_i < 0$$

Where  $T_j - T_i$  are the annual values in years  $j$  and  $i$ ,  $j > i$  respectively

If  $n < 10$ , the value of  $|S|$  is compared directly to the theoretical distribution of  $S$  derived by Mann and Kendall. The two tailed test is used. At certain probability level  $H_0$  is rejected in favour of  $H_1$  if the absolute value of  $S$  equals or exceeds a specified value  $S_{\alpha/2}$ , where  $S_{\alpha/2}$  is the smallest  $S$  which has the probability less than  $\alpha/2$  to appear in case of no trend. A positive (negative) value of  $S$  indicates an upward (downward) trend. For  $n \geq 10$ , the statistic  $S$  is approximately normally distributed with the mean and variance as follows  $E(S) = 0$

The variance ( $\sigma^2$ ) for the S- statistic is defined by

$$\sigma^2 = \frac{n(n-1)2n+5 - \sum ti(i-1)(2i+5)}{18}$$

In which  $t_i$  denotes the number of ties to extent  $i$ . The summation term in the numerator is used only if the data series

contains tied values. The standard test statistic  $Z_s$  is calculated as follows:

$$Z_s = \frac{S-1}{\sigma} \text{ for } S > 0$$

$$Z_s = 0 \text{ for } S = 0$$

$$Z_s = \frac{S+1}{\sigma} \text{ for } S < 0$$

The test statistic  $Z_s$  is used a measure of significance of trend. In fact, this test statistic is used to test the null hypothesis,  $H_0$ . If  $|Z_s|$  is greater than  $Z_{\alpha/2}$ , where  $\alpha$  represents the chosen significance level (e.g.: 5% with  $Z_{0.025} = 1.96$ ) then the null hypothesis is invalid implying that the trend is significant, (Motiee H 2009). Another statistic obtained on running the Mann-Kendall test is Kendall's tau, which is a measure of correlation and therefore measures the strength of the relationship between the two variables. Kendall's tau, like Spearman's rank correlation, is carried out on the ranks of the data.

That is, for each variable separately, the values are put in order and numbered, 1 for the lowest value, 2 for the next lowest and so on. In common with other measures of correlation, Kendall's tau will take values between  $\pm 1$  and  $+1$ , with a positive correlation indicating that the ranks of both variables increase together whilst a negative correlation indicates that as the rank of one variable increases, the other decreases. [Blackwell Publishing 2012] In time series analysis it is essential to consider autocorrelation or serial correlation, defined as the correlation of a variable with itself over successive time intervals, prior to testing for trends. Autocorrelation increases the chances of detecting significant trends even if they are absent and vice versa. In order to consider the effect of autocorrelation, Hamed and Rao (1998) suggest a modified Mann-Kendall test, which calculates the autocorrelation between the ranks of the data after removing the apparent trend. The adjusted variance is given by:

$$\text{Var}[S] = \frac{1}{18} [N(N-1)2N+5] \frac{n}{NS^*}$$

$$\text{Where } \frac{n}{NS^*}$$

$$= 1 + \frac{2}{N(N-1)(N-2)} \sum_{i=1}^p (N-i)(N-i-1)N-i-2)ps(i)$$

Where  $N$  is the number of observations in the sample,  $NS^*$  is the effective number of observations to account for autocorrelation in the data,  $ps(i)$  is the autocorrelation between ranks of the observations for lag  $i$ , and  $p$  is the maximum time lag under consideration (Sinha 2007). The null hypothesis is tested at 95% confidence level for Aerosol data for different regions in Chennai. The Statistical results are given in the Table 1.

On running the Mann-Kendall test PM 10 Aerosol for (Adayar, Anna Nagar T Nagar and Kilpauk) of Chennai Region daily data for the year 2013 to 2016 April, the following results were obtained. If the  $p$  value is less than the significance level  $\alpha$  (alpha) = 0.05,  $H_0$  is rejected. Rejecting  $H_0$  indicates that there is a trend in the time series, while accepting  $H_0$  indicates no trend was detected.

Table 1. Summary Statistics

Variable PM10/Region	Observation	Obs. With missing data	Obs. Without missing data	Minimum	Maximum	Mean	Std. deviation
Kilpauk	337	0	337	29	256	98.87	38.57
Anna Nagar	337	4	333	31	36	90.31	40.54
T.Nagar	450	116	334	38	341	103.95	36
Adayar	308	1	307	16	136	51.80	16.88

Table 2. Mann-Kendall trend test / Two-tailed test

Variable PM10/region	Kendall s tau	S	Var S	P value two tailed	alpha	Sen s slope	Test Interpretation
Kilpauk	-0.3330	-18767.00	10967398.15	.0001	.05	-0.0186	H <sub>0</sub> REJECTED
Anna Nagar	-0.2220	-12212.00	15518911.72	.0019	.05	-0.0159	H <sub>0</sub> REJECTED
T.Nagar	-0.1747	-9668.00	10092843.50	.0023	.05	-0.083	H <sub>0</sub> REJECTED
Adayar	-.0715	-3326.00	8857589.12	.2639	.05	-0.0186	H <sub>1</sub> REJECTED

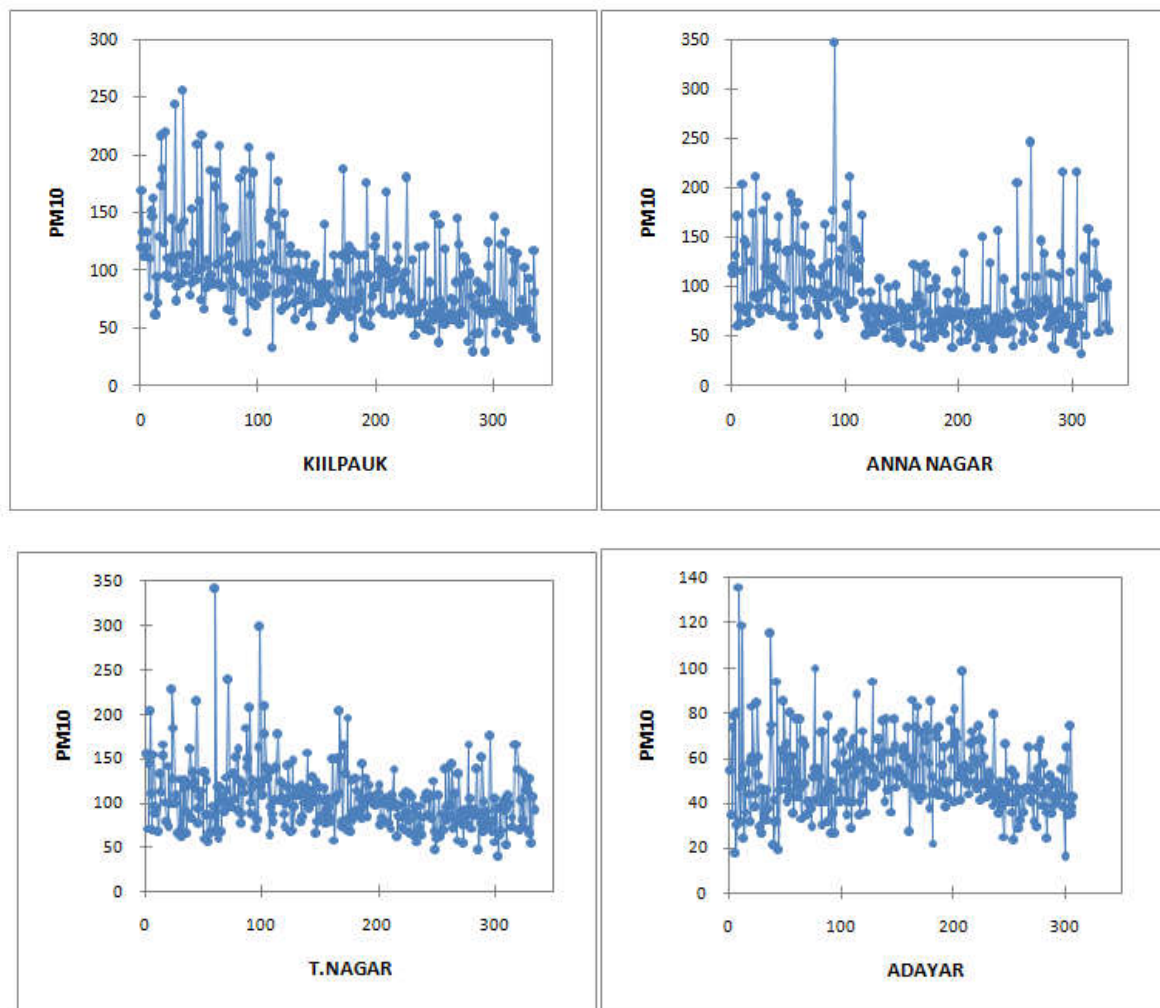


Figure 5. Plot of variation in aerosol for Chennai

On rejecting the null hypothesis, the result is said to be statistically significant. Table 1 indicates statistical results whereas the Table 2 give Mann-Kendall trend test / Two-tailed test. From the result is is observed that for the Chennai Region there is trend in PM 10for the regions Kilpauk, AnnaNagar and T.Nagar, though there was no trend seen in thePM 10 time series for the Adayar region. The aerosols play a significant role in the cloud microphysics. The negative value for Sens slope also indicates a negative trend in PM10.

## Conclusion

An attempt was made in this study to investigate the persistence behaviour and trend in the Aerosol time series. using Hurst Exponent and Man Kendall trend test .For this purpose Aerosol time series data of different regions of Chennai was taken into consideration. Using Hurst Exponent it was realised that the PM 10 data showed a completely persistence behaviour where as the trend analysis of PM10 using Mankendall showed trend in all the regions other than Adayar. Though any conclusion cannot be drawn at this

junction. The increasing trend is due to rapid urbanisation as cited in the literature. Better estimation of results could be obtained if the PM10 aerosol has sufficient available data points.

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