ISSN: 2230-9926

Available online at http://www.journalijdr.com

IJDR

International Journal of
DEVELOPMENT RESEARCH

International Journal of Development Research
Vol. 06, Issue, 06, pp. 8077-8084, June, 2016

Full Length Review Article

SILVER RATIOS, CARDANO'S FORMULA, AND VISUAL MODEL OF THE SECOND ATOMIC SHELL

***Alexander V. Yurkin**

City of Puschino, Russia

ARTICLE INFO

Article History:

Received 16th March, 2016
Received in revised form
02nd April, 2016
Accepted 24th May, 2016
Published online 30th June, 2016

Key Words:

Ray trajectories,
Mendeleev's periodic table,
Cardano's formula,
Atomic shells,
Triangles, silver houses.

ABSTRACT

This work is devoted to geometrical research of the ray trajectories system. We consider trajectories of electrons moving in the second shell of an atom. The proposed model of movement of electrons is based on the geometrical interpretation of the Pauli Exclusion Principle and the geometric interpretation of distribution of electrons in energy levels, shells, and subshells of atoms in Mendeleev's periodic table. It turned out that the second shell can be described by a cubic equation, Cardano's formula, and a new system of triangles as well. We present some new geometric ratios - silver ratios - between the elements of our model and the triangle system (the ratios between the sides, angles and areas of triangles). In the paper, we show that it is possible to describe atomic shells geometrically using complex numbers, and generalize our approach to geometric interpretation of the rest atomic shells. In the Appendix we apply the silver ratios to show the images of a silver parallelepiped and examples of silver houses and we compare our trajectories system with trajectories of car driving.

INTRODUCTION

Visual geometric models are used to describe various phenomena (Peters James F., *et al.*, 2015). Cardano's formula is used to find roots of a cubic equation to which various calculations, including the problems of trisection of an angle and doubling the cube well-known since ancient times, are reduced to. Unfortunately, the Academy of Sciences in Paris no longer considers projects related to these problems since the 18th century (Mathematical encyclopedic dictionary, 1988). In this paper, we use Cardano's formula to research recurrence relations in the geometrical model of the second electron shell of an atom (Yurkin A. V., 2016). For the convenience of readers, the Appendix hereto contains figures related to the nuclear topic taken from the previous publication of the author (Fig. 4) and from the well-known monograph in physics (Fig. 5 and 6). The possibility of using the silver ratios in other (non-nuclear) areas is shown in Fig. 4 captures and in Fig. 7 in the Appendix.

System of Trajectories

In papers (Yurkin A. V., 2015 and Yurkin A. V., 2016), we described a flat *system of periodic trajectories*. This system consists of ray groups, where the rays themselves are inclined at angles P narrow to the axis, which are multiples to the angle γ:

$P = (i + 1/2)\gamma, \quad i = 0, \pm 1, \pm 2 ....$ ……………………….(1)

Rays branching points are spaced apart horizontally at the distance L and vertically at small distances Q multiple to the length $k/2$:

$Q = jk/2, \qquad j = 0, \pm 1, \pm 2$ …………………………(2)

We called this ray system as $[P = \left(i + \frac{1}{2}\right)\gamma$ and $Q = jk/2]$ *system* or *semi-integer ray system*.

****Corresponding author: Alexander V. Yurkin***
City of Puschino, Russia

In Fig. 4 in the Appendix we provide an image of the second electron shell L as *[*$P = (i + 1/2)\gamma$, $Q = jk/2$*] system of periodic trajectories* from paper (Yurkin A. V., 2016). In paper (Yurkin A. V., 2016), it was shown that ray wavy trajectories correspond to atomic shells:

- Wavy trajectories containing segments inclined at angles $\gamma/2$ to the horizontal correspond to the first electron shell of an atom;
- Wavy trajectories containing segments inclined at angles $\gamma/2$ and $3\gamma/2$ to the horizontal correspond to the second electron shell of an atom;
- Wavy trajectories containing segments inclined at angles $\gamma/2$, $3\gamma/2$, and $5\gamma/2$ to the horizontal correspond to the third electron shell of an atom;
- Wavy trajectories containing segments inclined at angles $\gamma/2$, $3\gamma/2$, $5\gamma/2$, $7\gamma/2$ and to the horizontal correspond to the third electron shell of an atom; and so on.

To illustrate coherence of our visual geometric model with the known approaches to description of an atom, Fig. 5 in the Appendix shows atomic shells and subshells, and Fig. 6 in the Appendix shows a part of the periodic table from monograph (Savelyev I. V., 1982). Fig. 1 shows the completely filled second electron shell L as a *system of periodic trajectories* (equations 1 and 2). The inclination angles of the trajectory segments are $\gamma/2$ and $3\gamma/2$. $\mathcal{E}$ is the energy propagating along the trajectory segment ($\mathcal{E}$ is the weight / weight factor).

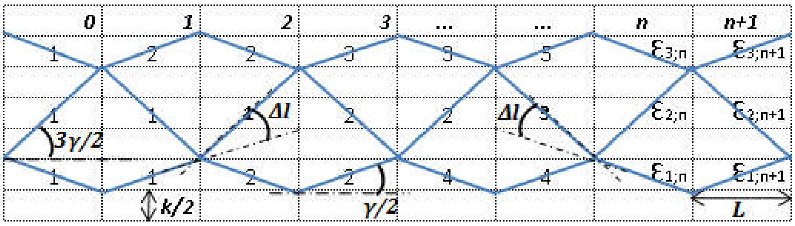

Fig.1. System of periodic (wavy) trajectories

The second electron shell is filled completely. Quantum transitions l are shown in the form of angles between dash-dotted lines. n is the number of passes (iterations). $\mathcal{E}_{1;n}$, $\mathcal{E}_{2;n}$, and $\mathcal{E}_{2;n}$ is the energy propagating along the segments of trajectories $1, 2,$ and 3 within the same pass n. The segments are numbered upwards.

In this case, as the rays propagate along the branching segments, energy is added as follows:

$$\left.\begin{aligned} \mathcal{E}_{1;n+1} &= \mathcal{E}_{1;n} \\ \mathcal{E}_{2;n+1} &= \mathcal{E}_{3;n} \\ \mathcal{E}_{3;n+1} &= \mathcal{E}_{2;n} + \mathcal{E}_{3;n} \end{aligned}\right\} \qquad \text{..................(3)}$$

Paper (Yurkin A. V., 1994) showed that propagation of rays at angles $\gamma/2$ and $3\gamma/2$ in such a system (Fig. 1) was described by sequences:

$$\left.\begin{aligned} &2, 3, 4, 5, 7, 9, 12, 16, 21, 28, \ldots \text{ - for } \gamma/2 \text{ and} \\ &1, 1, 1, 2, 2, 3, \ 4, \ 5, \ 7, \ 9, \ldots \text{ - for } 3\gamma/2. \end{aligned}\right\} \qquad \text{..................(4)}$$

Members of these sequences are expressed by the following recurrent formula:

$$u_n = u_{n-2} + u_{n-3}. \qquad \text{................................(5)}$$

Where:

$$u_n = \mathcal{E}_{1;n} + \mathcal{E}_{3;n}, \text{ and } u_{n-4} = \mathcal{E}_{2;n}. \qquad \text{.............................. (6)}$$

Relative distribution of energy $\mathcal{K}$ at the angle (along the angles $\gamma/2$ and $3\gamma/2$) is as follows:

$$\mathcal{K} = \frac{u_n}{u_{n-4}} = \frac{\mathcal{E}_{1;n}+\mathcal{E}_{3;n}}{\mathcal{E}_{2;n}}. \qquad \text{................................(7)}$$

The value of $\mathcal{K}$ is numerically equal to the relative size of the quantum transition l between the first and the second energy levels of the shell L:

$$\mathcal{K} = l, \qquad \text{...........................(8)}$$

The ratio of the number of rays $\mathcal{K}$ (the ratio of energies propagating along the trajectory segments) within the same pass in Fig. 1 can be calculated accurately. If the number of passes n is infinite, it is expressed in radicals. It can be found by consideration of the geometric progression:

$$1, q, q^2, q^3, \qquad \text{…………………………(9)}$$

It similar to consideration of Fibonacci sequence (Vorob'ev N. N., 1992). For progression (9) to be the solution of equation (5), the following ratio shall be met for any n:

$$q^n = q^{n-2} + q^{n-3}.$$

Hence, upon dividing by q^{n-3}, we get:

$$q^3 = q + 1. \qquad \text{………………………..(10)}$$

The real root q of cubic equation (10) is calculated by Cardano's formula:

$$q_1 = v + w = 1{,}3246. \qquad \text{…………………..(11)}$$

Where (in our case): $v = \sqrt[3]{\frac{1}{2} + \sqrt{\frac{1}{4}\ \ \frac{1}{27}}} \approx 0{,}9869$ and $w = \sqrt[3]{\frac{1}{2}\ \ \sqrt{\frac{1}{4}\ \ \frac{1}{27}}} \approx 0{,}3377.$

Therefore, for a steady-state distribution we have expression (7) as follows:

$$\mathcal{K} = \frac{u_n}{u_{n-4}} = q_1^4 \approx 3{,}0796 \qquad \text{……………………(12)}$$

Silver Ratios

By analogy with the ratios for Fibonacci numbers and the *golden ratio* (Vorob'ev N. N., 1992) (golden ratio equation $p^2 = p + 1$), let us write down separately some ratios between real numbers for our cubic case (5, 6, and 10):

$$q_1 = \frac{u_{n+1}}{u_n} = \sqrt[4]{\frac{u_{n+4}}{u_n}}. \qquad \text{………………………..(13)}$$

Given that $\mathcal{E}_{1;n} = \mathrm{a};\ \mathcal{E}_{2;n} = \mathrm{b};$ and $\mathcal{E}_{3;n} = \mathrm{c}$, we have:

$$q_1 = \sqrt{\frac{a}{b}}, \qquad \text{……………………………..(14)}$$

$$q_1 = \sqrt[4]{\frac{a+c}{b}}, \qquad \text{……………………………... (15)}$$

$$q_1 = \frac{b+c}{a}, \qquad \text{……………………………… (16)}$$

$$q_1 = \frac{c}{b} = \frac{b}{a-b}, \qquad \text{……………………………… (17)}$$

$$q_1 = \frac{a}{c}, \qquad \text{…………………………………..(18)}$$

$$q_1 = \frac{b}{a} + \frac{b}{c}, \qquad \text{…………………………………..(19)}$$

$$\mathcal{K} = \frac{a^2}{b^2}, \text{ and so on.} \qquad \text{………………………………… (20)}$$

Equations (14 to 20) can be proved numerically or by successive substitutions until we arrive at equation (10).

Normalised form:

$$a + b + c = 1, \qquad \text{……………………………...(21)}$$

$$\frac{a}{b}\ \ \frac{c}{a} = 1. \qquad \text{………………………………..(22)}$$

Where $a \approx 0{,}4302$; $b \approx 0{,}2451$; and $c \approx 0{,}3247$.

Formula (11) may be written down as follows:

$$q_1 = v + w = c + 1 = a + b + 2c \qquad \text{.......................... (23)}$$

We shall take q_1 as the principal number of *silver ratios*, and a, b, and c as additional numbers of silver ratios.

Some Generalisations

Except for one real root, let us write two complex roots also received from Cardano's formula:

$$q_{2,3} = \frac{v+w}{2} \pm \frac{v-w}{2}\sqrt{3},\ \text{where} \ \ = \sqrt{\ \ 1}. \qquad \text{........................ (24)}$$

$$\text{And}\ \left(\frac{v+w}{2}\right)^2 + \left(\frac{v-w}{2}\sqrt{3}\right)^2 = \frac{1}{v+w} = \frac{1}{q_1} = \frac{b}{c} = \frac{c}{a} = a + c = \sqrt[4]{c}, \qquad \text{.........................(25)}$$

where $\left(\frac{v+w}{2}\right)^2 \approx 0{,}4387 \neq a$, and $\left(\frac{v-w}{2}\sqrt{3}\right)^2 \approx 0{,}3162 \neq$ c.

$$\text{And}\ \ v, w = \frac{q_1}{2} \pm \sqrt{\frac{{q_1}^2}{4} \ \ \frac{1}{3}},\ \text{and so on.} \qquad \text{...................................(26)}$$

We shall consider expressions (24) as complex ratios between the values of the electron energy levels of our atomic model expressed by complex numbers. Note that all the three roots $q_{1,2,3}$ of equation (10) can also be shown in a three-dimensional geometric ray model, e.g., as we did it in paper (Popyrin *et al*., 1999).

It can be assumed that if the trajectory systems (1 and 2) contain segments inclined at the angles $\frac{\gamma}{2}, \frac{3\gamma}{2}, \frac{5\gamma}{2}, \frac{7\gamma}{2}$, ..., then these systems can be described by equations of the first, third, fifth, seventh, etc. degrees, respectively, similar to equation (10).

Silver Triangles

To represent the above ratios visually, we shall present numbers of the silver ratios a, b, c, and q_1 as parts of a triangle (let us call it a *silver triangle* for our cubic case, by analogy with the *golden triangle* for the flat case (Vorob'ev N. N., 1992)) and systems of similar triangles.

Fig. 2. Shows such a triangle

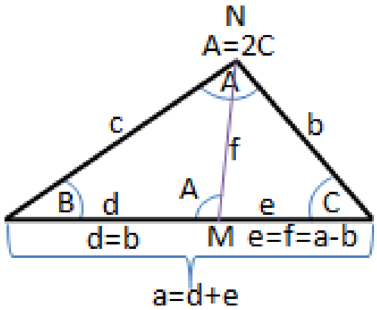

Fig. 2. Silver Triangle with sides a, b, and c, angles A, B, and C, and bisector f of the angle A.

Ratios of the sides in this triangle are expressed by equations (16 to 23). The bisector f of the angle A divides the opposite side of the triangle into parts as follows:

$a = d + e$, where $d = b$ and $e = f$.

Perimeter $\mathcal{P}$ of the triangle is expressed by equation (21):

$\mathcal{P} = a + b + c = 1.$

Therefore, formula (23) may also be written down as follows:

$q_1 = \mathcal{P} + c.$

Ratios between the angles in this triangle are as follows:

$$\left.\begin{aligned} A &= 2C = cos^{-1}(\ \tfrac{b}{2}) \approx 1{,}6937 \approx 97{,}0399^{\circ} \\ B &= \pi \quad 3C = cos^{-1}(\tfrac{2c+1}{2}) \approx 0{,}6011 \approx 34{,}4401^{\circ} \\ C &= cos^{-1}(\tfrac{c+1}{2}) = cos^{-1}(\tfrac{q_1}{2}) \approx 0{,}8468 \approx 48{,}5200^{\circ}. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

Ratios of the areas S of triangles ABC, BNM, and CNM are as follows:

$$\frac{S_{ABC}}{S_{BNM}} = q_1;\ \frac{S_{ABC}}{S_{CNM}} = {q_1}^4 + 1;\ \text{and}\ \frac{S_{BNM}}{S_{CNM}} = {q_1}^4. \quad \ldots\ldots\ldots\ldots\ldots..(28)$$

We can obtain from (28) a quantic equation similar to (coincided with) cubic equation (10):

$${q_1}^5 = {q_1}^4 + 1. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(29)$$

The image of the system of similar triangles in Fig. 3 gives us additional visual presentation of silver ratios:

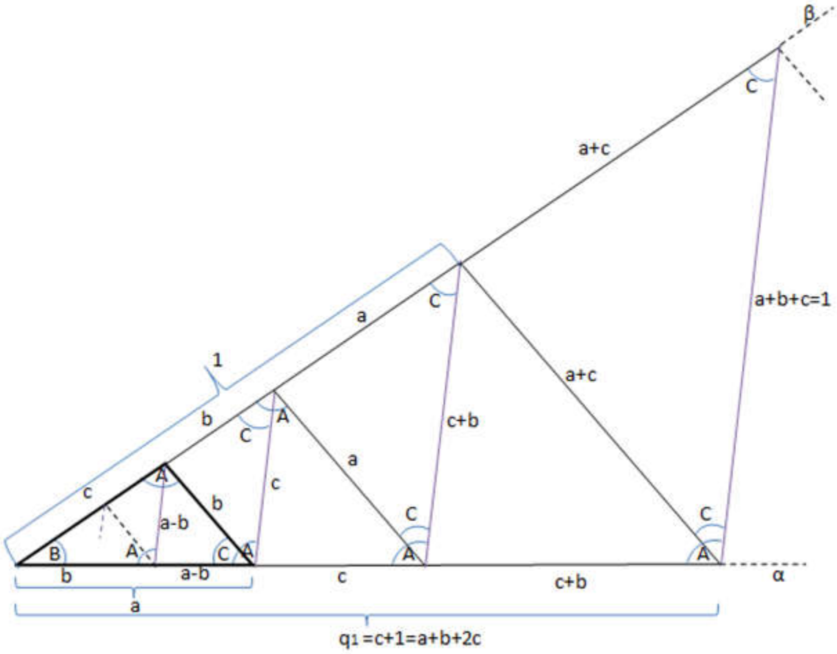

Fig. 3. System of triangles similar to the silver triangle

The system of similar triangles is located between two lines α and β, where B is the angle between them. The parallel lines are crossing the lines α and β at the angles A and C. The points of intersection of the parallel lines and the line α at the angles A and C converge. The point of intersection of the parallel lines and the line β at the angles C and A converge too. Thus, the parallel lines form a zigzag between the lines α and β. The ratios between pieces of the figure shown in Fig. 3 are obvious.

Conclusion

The atomic structure is usually described using the periodic table approach based on Mendeleev's system of chemical elements (Savelyev I. V., 1982 and Landau L. D. and Livshits E. M., 1989), or non-visual mathematical formalistic approach based on the Bohr-Sommerfeld theory and quantum mechanics (Savelyev I. V., 1982, Sommerfeld Arnold, 1973, and Landau L. D. and Livshits E. M., 1989). The mathematical formalistic approach is used in other fields of physics for example for laser beams investigations (Mensky M. B. and Yurkin A. V., 2008). In our paper, we used a geometric approach based on research of visual geometric models in the form of trajectories and triangles. We hope that our visual geometric approach can complement the above two approaches to contribute in research of the atomic structure in its entire complexity.

Acknowledgements

The author expresses gratitude to Prof. S. E. Shnoll, Prof. V. V. Dikusar, Prof. V. V. Smolyaninov, Prof. E. I. Mayevsky, Prof. V. G. Mikhalevich, Dr. D. E. Shnoll, Dr. A. Shemetov, Dr. O. Nersesyan and Dr. S. Sakhaeva for useful discussions. The author also thanks participants of a seminar of theoretical and experimental biophysics of the city of Puschino which has taken place under the leadership of Dr. O. Mornev for new ideas.

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Appendix

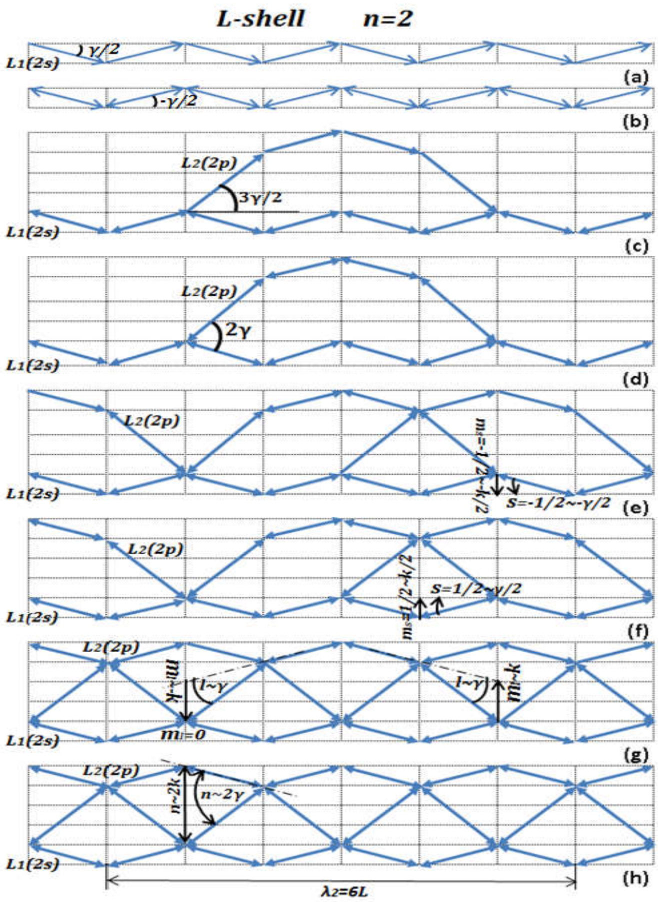

Fig.4. One of the eight ray groups of $\{P = (i + 1/2)\gamma$ and $Q = jk/2\}$ subsystem of *periodic trajectories* and the sequential process of filling the shell L. n is the principal quantum number, l is the orbital quantum number, m_l is the magnetic one, and m_s (or s) is the spin quantum number. The dash-doted lines show the reference axes for measuring the angles and distances. (Fig. 15 from (Yurkin A. V., 2016)). We will give briefly in addition an example not from nuclear area where it is possible to observe the

shown trajectories. Such trajectories can arise at the movement of the single car on a direct road. The driver turns a wheel once on small angles $\pm\gamma$ from time to time; so that the inclination angles of the trajectory segments are $\pm\gamma/2$ to the axis of road (a, b). The driver turns a wheel once or twice on small angles $\pm\gamma$ from time to time; so that the inclination angles of the trajectory segments are $\pm\gamma/2$ or $\pm3\gamma/2$ to the axis of road (c – h).

Shell	n	l	m_l	m_s	Subshell	Shell	n	l	m_l	m_s	Subshell
K	1	0	0	↑↓	$K(1s)$	N	4	0	0	↑↓	$N_1(4s)$
L	2	0	0	↑↓	$L_1(2s)$			1	−1 0 +1	↑↓ ↑↓ ↑↓	$N_2(4p)$
		1	−1 0 +1	↑↓ ↑↓ ↑↓	$L_2(2p)$			2	−2 −1 0 +1 +2	↑↓ ↑↓ ↑↓ ↑↓ ↑↓	$N_3(4d)$
M	3	0	0	↑↓	$M_1(3s)$			3	−3 −2 −1 0 +1 +2 +3	↑↓ ↑↓ ↑↓ ↑↓ ↑↓ ↑↓ ↑↓	$N_4(4f)$
		1	−1 0 +1	↑↓ ↑↓ ↑↓	$M_2(3p)$						
		2	−2 −1 0 +1 +2	↑↓ ↑↓ ↑↓ ↑↓ ↑↓	$M_3(3d)$						

Fig.5. Division of possible states of an electron in an atom into shells and subshells (Table 36.1 from (Savelyev I. V., 1982))

Element	K	L		M			N		Basic therm
	1s	2s	2p	3s	3p	3d	4s	4p	
1 H	1	—	—	—	—	—	—	—	$^2S_{1/2}$
2 He	2	—	—	—	—	—	—	—	$^1S_0$
3 Li	2	1	—	—	—	—	—	—	$^2S_{1/2}$
4 Be	2	2	—	—	—	—	—	—	$^1S_0$
5 B	2	2	1	—	—	—	—	—	$^2P_{1/2}$
6 C	2	2	2	—	—	—	—	—	$^3P_0$
7 N	2	2	3	—	—	—	—	—	$^4S_{3/2}$
8 O	2	2	4	—	—	—	—	—	$^3P_2$
9 F	2	2	5	—	—	—	—	—	$^2P_{3/2}$
10 Ne	2	2	6	—	—	—	—	—	$^1S_0$
11 Na	2	8		1	—	—	—	—	$^2S_{1/2}$
12 Mg	2	8		2	—	—	—	—	$^1S_0$
13 Al	2	8		2	1	—	—	—	$^2P_{1/2}$
14 Si	2	8		2	2	—	—	—	$^3P_0$
15 P	2	8		2	3	—	—	—	$^4S_{3/2}$
16 S	2	8		2	4	—	—	—	$^3P_2$
17 Cl	2	8		2	5	—	—	—	$^2P_{3/2}$
18 Ar	2	8		2	6	—	—	—	$^1S_0$
19 K	2	8		8		—	1	—	$^2S_{1/2}$
20 Ca	2	8		8		—	2	—	$^1S_0$
21 Sc	2	8		8		1	2	—	$^2D_{3/2}$
22 Ti	2	8		8		2	2	—	$^3F_2$
23 V	2	8		8		3	2	—	$^4F_{3/2}$
24 Cr	2	8		8		5	1	—	$^7S_3$
25 Mn	2	8		8		5	2	—	$^6S_{5/2}$
26 Fe	2	8		8		6	2	—	$^5D_4$
27 Co	2	8		8		7	2	—	$^4F_{9/2}$
28 Ni	2	8		8		8	2	—	$^3F_4$
29 Cu	2	8		8		10	1	—	$^2S_{1/2}$
30 Zn	2	8		8		10	2	—	$^1S_0$
31 Ga	2	8		8		10	2	1	$^2P_{1/2}$
32 Ge	2	8		8		10	2	2	$^3P_0$
33 As	2	8		8		10	2	3	$^4S_{3/2}$
34 Se	2	8		8		10	2	4	$^3P_2$
35 Br	2	8		8		10	2	5	$^2P_{3/2}$
36 Kr	2	8		8		10	2	6	$^1S_0$

Fig.6. The process of filling the electron shells of the first 36 elements of the periodic system (Table 37.1 from (Savelyev I. V., 1982))

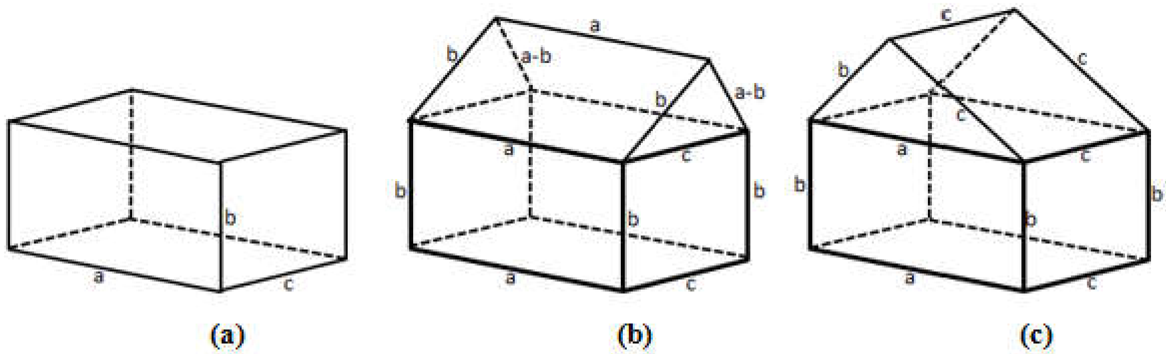

Fig.7. Silver parallelepiped (a) and silver houses (b) and (c)

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