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TRANSIENT MHD MASS TRANSFER FLOW PAST AN IMPULSIVELY STARTED VERTICAL PLATE IN A POROUS MEDIUM WITH RAMPED TEMPERATURE, THERMAL DIFFUSION AND RADIATION

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ABSTRACT

In this paper we discuss, an exact solution to the problem of a MHD viscous, incompressible free convective flow of an electrically conducting, Newtonian non Grey fluid past a suddenly started infinite vertical plate with ramped wall temperature and concentration in presence of appreciable radiation heat transfer and uniform transverse magnetic field is presented. The fluid is assumed negligible magnetic Reynolds number considered Induced hydro magnetic effects. The equations governing the flow are solved by using Laplace transform technique in closed form. The influence of Hartmann number, radiation parameter, Darcy number, Reynolds number and time on the variations in the fluid velocity, fluid temperature, fluid concentration, skin friction, Nusselt number and Sherwood number at the plate are demonstrated graphically and physically interpreted.

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INTRODUCTION

Many Engineering problems are susceptible to MHD analysis. The study of flow problems has achieved remarkable interest due to its application in MHD generators, MHD pumps and MHD flow meters etc. MHD principles find its applications in Medicine and Biology also the study of effects of magnetic field on free convection flow is important in liquid metals, electrolytes and ionized gases. Geophysics encounters MHD phenomena in interaction on conducting fluids and magnetic fields. The rotating flow of an electrically conducting fluid in presence of magnetic field has got its importance in Geophysical problems. The present form of MHD is due to the pioneer contributions of several notable authors like Alfven (1942), Cowling (1957), Shercliff (1965), Ferraro and Plumpton (1966) and Crammer and Pai (1978). The natural flow arises in fluid when the temperature change causes density variation leading to buoyancy forces acting on the fluid. Free convection is a process of heat transfer in natural flow. Radiation is another very important factor which plays very significant role in the free convection process that occurs due to temperature difference. Radiative convective flows are encountered in many environmental and industrial processes. So studies on effect of radiation on free convective fluid flows have been drawing great interest among the researchers since long back to present. Gupta and Gupta (1974) [6] studied the effect of radiation on combined free and forced convection of an electrically conducting fluid flow through an open ended vertical channel in presence of uniform magnetic field. Radiation is another process of heat transfer through electromagnetic waves. Radiative convective flows are encountered in countless industrial and environment processes like heating and cooling chambers, evaporation from large open water reservoirs, astrophysical flows and solar power technology.

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Due to importance of the above physical aspects, several authors have carried out model studies on the problems of free convective flows of incompressible viscous fluid under different flow geometries taking into account of the thermal radiation. Some of them are Mansour (1990), Raptis and Perdikis (1999), Ganesan and Loganathan (2002), Mbeledogue et al. (2007), Makinde (2005) and Sattar and Kalim (1996). Investigation of problems on natural convective radiating flow of electrically conducting fluid past an infinite plate becomes very interesting and fruitful when a magnetic field is applied normal to the plate. Comprehensive literature on various aspects of free convective radiative MHD flows and its applications can be found in Sattar and Maleque(2000), Samad and Rahman(2006), Orhan and Ahmet (2008), Prasad et al. (2006), Takhar et al.(1996), Ahmed and Sarmah(2009) and Ahmed (2012). The effect of rotation on unsteady hydro magnetic natural convection flow of a viscous incompressible electrically conducting fluid past an impulsively moving vertical plate with ramped wall temperature has been investigated recently by Seth et al. (2011). Very recently, Das et al. (2011) have studied the radiation effect on natural flow of an optically thin viscous incompressible fluid near a suddenly moving vertical plate with ramped wall temperature by adopting Cogley-Vincentine-Gilles equilibrium model (1968). By optically thin, it is meant that the fluid has no self absorption property, but it can absorb radiation emitted by boundaries.

Recently Ahmed and Dutta investigated the effect of a uniform transverse magnetic field on a natural flow of an optically thin electrically conducting viscous incompressible radiating non-Grey fluid (Cogley (1968)) past an impulsively started vertical plate with temporary ramped wall temperature. In the present work an attempt has been made to study the combined effect of mass diffusion and thermal diffusion on the natural flow of an optically thin electrically conducting viscous incompressible radiating non-Grey fluid past an impulsively started vertical plate with temporary ramped wall temperature and ramped concentration. As a magnetic field of uniform strength is assumed to be applied transversely, the medium is considered to be homogeneous due to MHD convection in a porous media emphasized by Nield (2008). In this work, characteristic time t_0 is variable the Reynolds number Re comes to existence explicitly and hence the influence of the Reynolds number on the flow field, G_r Grashof number of heat transfer in addition to other similarity parameters is also to be investigated in the of our study.

MATHEMATICAL FORMULATION

The equations governing the motion of an incompressible, viscous, electrically conducting radiating fluid past solid surface in presence of a magnetic field are:

$$\text{Continuity equation: } \bar{\nabla} \cdot \bar{q} = 0 \quad (1)$$

$$\text{Magnetic field continuity equation: } \bar{\nabla} \cdot \bar{B} = 0 \quad (2)$$

$$\text{Ohm's law for moving conductor: } \bar{J} = \sigma (\bar{E} + \bar{q} \times \bar{B}) \quad (3)$$

$$\text{Momentum equation: } \rho \left[\frac{\partial \bar{q}}{\partial t'} + (\bar{q} \cdot \bar{\nabla}) \bar{q} \right] = -\bar{\nabla} p + \bar{J} \times \bar{B} + \rho \bar{g} + \mu \nabla^2 \bar{q} \quad (4)$$

$$\text{Energy equation: } \rho C_p \left[\frac{\partial T'}{\partial t'} + (\bar{q} \cdot \bar{\nabla}) T' \right] = k \nabla^2 T' + \varphi + \frac{J^2}{\sigma} + \frac{\partial q_r}{\partial n'} \quad (5)$$

$$\text{Species continuity equation: } (\bar{q} \cdot \bar{\nabla}) \bar{C} = D_M \nabla^2 \bar{C} + D_T \nabla^2 \bar{T} \quad (6)$$

All the physical quantities are defined in the Nomenclature.

We now consider an unsteady radiative MHD free convective flow of an incompressible viscous and electrically conducting optically thin non-Grey fluid past a suddenly moving infinite vertical plate in its own place with temporary ramped temperature in presence of magnetic field of uniform strength B_0 applied normal to the plate directed into fluid region.

Our investigation is restricted to the following assumption:

- All the fluid properties are considered constants except the influence of the variation in density in the buoyancy force term.
- The viscous dissipation is also assumed to be negligible in the energy equation as the motion is due to free convection only.
- The magnetic Reynolds number is so small for that the induced magnetic field can be neglected in comparison to the applied magnetic field.
- The plate is electrically non-conducting.
- The radiation heat flux in the direction of the plate velocity is considered negligible in comparison to that in the normal direction.
- No external field is applied for which the polarization voltage is negligible leading to $\bar{E} = \bar{0}$

Initially the plate and surrounding fluid were at rest at the same temperature T'_∞ and Concentration C'_∞ . At time $t' > 0$, the plate is suddenly moved in its own plane with a Constant velocity U_0 and the temperature and concentration of the wall is raised to $T'_\infty + (T'_w - T'_\infty) \frac{t'}{t_0}$ and $C'_\infty + (C'_w - C'_\infty) \frac{t'}{t_0}$ for $0 < t' \leq t_0$ and the Constant temperature $T'_w (T'_w > T'_\infty)$ and concentration $C'_w (C'_w > C'_\infty)$ is maintained at $t' > t_0$. We now introduce a coordinate system (x', y', z') With X-axis along the plate in the upward Direction, Y-axis normal to the plate directed in the fluid region Z-axis along the width of plate. Let $q = (u', 0, 0)$ denote the fluid velocity and $\bar{B} = (0, B_0, 0)$ be the applied magnetic field at the Point (x', y', z', t') in the fluid.

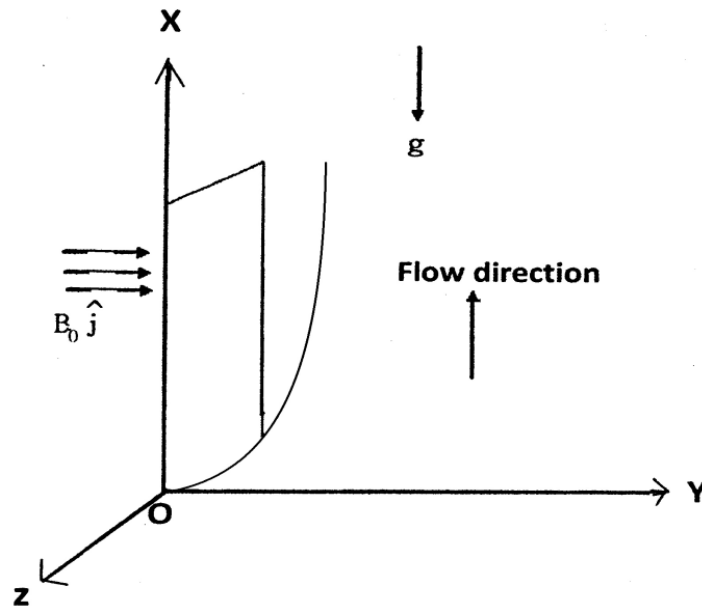


Figure 1. Physical model

With the foregoing assumptions and under the usual boundary layer and Boussinesq Approximations, the equations (1), (4), (5) and (6) reduce to

$$\frac{\partial u'}{\partial x'} = 0, \text{ which yields } u' = u'(y', t') \quad (7)$$

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g \beta (T' - T'_\infty) + g \bar{\beta} (C' - C'_\infty) - \frac{\sigma B_0^2}{\rho} u' - \frac{\nu}{k} u' \quad (8)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + \frac{\partial q_r}{\partial y'} \quad (9)$$

$$\frac{\partial C'}{\partial t'} = D_M \frac{\partial^2 C'}{\partial y'^2} + D_T \frac{\partial^2 T'}{\partial y'^2} \quad (10)$$

The appropriate initial and boundary conditions are

$$u' = 0, \quad T' = T'_\infty, C' = C'_\infty \quad \forall y', \quad t' \leq 0 \quad (11)$$

$$u' = U_0, \quad T' = T'_\infty + \frac{T'_w - T'_\infty}{t_0} t', \quad C' = C'_\infty + \frac{C'_w - C'_\infty}{t_0} t' \quad \text{at } y' = 0, \quad 0 < t' \leq t_0 \quad (12)$$

$$u' = U_0, \quad T' = T'_w, \quad C' = C'_w \quad \text{at } y' = 0, \quad t' > t_0 \quad (13)$$

$$u' \rightarrow 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \quad \text{at } y' \rightarrow \infty, \quad t' > 0 \quad (14)$$

It is emphasized by Cogley et al. (1968) that the rate of radiative flux in optically thin limit for a non-Grey gas near equilibrium is given by

$$\frac{\partial q_r}{\partial y'} = 4I (T' - T'_\infty) \quad (15)$$

Where, $I = \int_0^\infty (K_\lambda)_w \left(\frac{\partial e_{b\lambda}}{\partial T'}\right)_w d\lambda$

On use of (15), (9) reduce to $\frac{\partial T'}{\partial t'} = \alpha \frac{\partial^2 T'}{\partial y'^2} - F(T' - T_\infty)$ (16)

Proceeding with the analysis, we introduce the following non-dimensional variables and similarity parameters to normalize the flow model:

$$u = \frac{u'}{U_0}, y = \frac{y'}{U_0 t_0}, t = \frac{t'}{t_0}, Gr = \frac{vg\beta(T'_w - T'_\infty)}{U_0^2}, G_m = \frac{vg\bar{\beta}(C'_w - C'_\infty)}{U_0^2}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}$$

$$= \frac{C' - C'_\infty}{C'_w - C'_\infty}, Pr = \frac{\mu C_p}{k}, Q = \frac{4Iv}{\rho C_p U_0^2}, M = \frac{\sigma B_0^2 v}{\rho U_0^2}, Re = \frac{U_0^2 t_0}{\nu}$$

$$Sr = \frac{DT(T'_w - T'_\infty)}{v(C'_w - C'_\infty)}, Sc = \frac{v}{D_M}, \alpha = \frac{k}{\rho C_p}, F = \frac{4I\theta}{\rho C_p}$$
 (17)

All the physical quantities are defined in the Nomenclature.

By virtue of transformations cum definitions (17), the equations (8), (9) and (10) in normalized form become

$$\frac{\partial u}{\partial t} = \frac{1}{Re} \frac{\partial^2 u}{\partial y^2} + ReGr\theta + ReGm - MReu$$
 (18)

$$\frac{\partial \theta}{\partial t} = \frac{1}{PrRe} \frac{\partial^2 \theta}{\partial y^2} - QRe$$
 (19)

$$\frac{\partial}{\partial t} = \frac{1}{ScRe} \frac{\partial^2}{\partial y^2} + \frac{Sr}{Re} \frac{\partial^2 \theta}{\partial y^2}$$
 (20)

Subject to relevant initial and boundary conditions:

$$u=0, \theta=0, \bar{u}=0 \quad \forall y \geq 0 \text{ and } t \leq 0$$
 (21)

$$u=1, \theta=t, \bar{u}=t \quad \text{at } y=0, \quad 0 < t \leq 1$$
 (22)

$$u=1, \theta=1, \bar{u}=1 \quad \text{at } y=0, \quad t > 1$$
 (23)

$$u=0, \theta=0, \bar{u}=0 \quad \text{as } y \rightarrow \infty, \quad t > 0$$
 (24)

METHOD OF SOLUTION

On taking Laplace Transform of the equations (18),(19)and (20),the combined initial and boundary value problem reduce to a boundary value problem governed by the equations

$$\frac{d^2 \bar{u}}{dy^2} - (MRe^2 + sRe) \bar{u} = R e^2 (Gr\bar{\theta} + Gm^-)$$
 (25)

$$\frac{d^2 \bar{\theta}}{dy^2} - (s + QRe) PrRe\bar{\theta} = 0$$
 (26)

$$\frac{d^2 \bar{u}}{dy^2} + ScSr \frac{d^2 \bar{\theta}}{dy^2} - sSc Re^- = 0$$
 (27)

Subject to the boundary conditions

$$\bar{u} = \frac{1}{s}, \quad \bar{\theta} = \frac{1}{s^2}(1 - e^{-s}), \quad \bar{u}^- = \frac{1}{s^2}(1 - e^{-s}) \text{ at } y=0$$
 (28)

$$\bar{u}=0, \quad \bar{\theta}=0, \quad \bar{u}^- = 0 \text{ at } y \rightarrow \infty$$
 (29)

The equations (25)-(27) are two ordinary coupled second order differential equations and the solutions of these equations subject to the conditions (28) and (29) are as follows:

$$\bar{\theta} = \frac{1}{s^2}(1 - e^{-s})e^{-\sqrt{PrRe(QRe+s)}y}$$
 (30)

$$\begin{aligned}
&= \left[\frac{A_8}{s^2} (1 - e^{-s}) - A_3 \left(\frac{A_4}{s} + \frac{A_5}{s+B_3} \right) (1 - e^{-s}) \right] e^{-\sqrt{scResy}y} + \\
&A_7 \frac{1}{s^2} (1 - e^{-s}) e^{-\sqrt{Pr(s+QRe)y}y} + A_3 \left[\left(\frac{A_4}{s} + \frac{A_5}{s+B_3} \right) (1 - e^{-s}) e^{-\sqrt{PrRe(s+QRe)y}y} \right] \quad (31) \\
\bar{u} &= \left[\frac{1}{s(1-e^{-s})} \quad A_{46} \frac{1}{s} \quad A_{39} \frac{1}{s+A_{15}} \quad A_{47} \frac{1}{s^2} \quad A_{48} \frac{1}{s+B_3} \quad A_{43} \frac{1}{s+A_{17}} \right] \\
&(1 - e^{-s}) e^{-\sqrt{Re(s+MRe)y}y} + A_{38} \left[\frac{1}{s} (1 - e^{-s}) e^{-\sqrt{PrRe(s+QRe)y}y} \right] \\
&+ A_{39} \left[\frac{1}{s+A_{15}} (1 - e^{-s}) \right] e^{-\sqrt{PrRe(s+QRe)y}y} + A_{40} \left[\frac{1}{s^2} (1 - e^{-s}) \right] e^{-\sqrt{PrRe(s+QRe)y}y} \\
&+ A_{41} \left[\frac{1}{s+A_{15}} (1 - e^{-s}) \right] e^{-\sqrt{PrRe(s+QRe)y}y} + A_{42} \left[\frac{1}{s} (1 - e^{-s}) \right] \\
&+ A_{41} \left[\frac{1}{s+B_3} (1 - e^{-s}) \right] e^{-\sqrt{PrRe(s+QRe)y}y} + e^{-\sqrt{scResy}y} + A_{43} \left[\frac{1}{s+A_{17}} (1 - e^{-s}) \right] \\
&e^{-\sqrt{scResy}y} + A_{44} \left[\frac{1}{s^2} (1 - e^{-s}) \right] e^{-\sqrt{scResy}y} + A_{45} \left[\frac{1}{s+B_3} (1 - e^{-s}) \right] e^{-\sqrt{scResy}y} \quad (32)
\end{aligned}$$

Taking inverse Laplace Transforms of the equations (30), (31) and (32) we have

$$\theta = \Psi_1 \Psi_2 \quad (33)$$

$$\begin{aligned}
\phi &= A_8 (\Psi_3 \Psi_4) \quad A_3 A_4 (\Psi_5 \Psi_6) \quad A_3 A_5 e^{-B_3 t} (\Psi_7 \quad e^{-B_3} \Psi_8) + A_7 (\Psi_1 \Psi_2) \\
&+ A_3 A_4 (\Psi_9 \Psi_{10}) + A_3 A_5 e^{-B_3 t} (\Psi_{11} \quad e^{B_3} \Psi_{12}) \quad (34)
\end{aligned}$$

$$\begin{aligned}
U &= \Psi_{13} \quad A_{46} (\Psi_{13} \Psi_{14}) \quad A_{39} e^{-A_{15} t} (\Psi_{15} \quad e^{-A_{15}} \Psi_{16}) \quad A_{47} (\Psi_{17} \Psi_{18}) - \\
&A_{48} e^{-B_3 t} (\Psi_{19} \quad e^{-B_3} \Psi_{20}) \quad A_{43} e^{-A_{17} t} (\Psi_{21} \quad e^{-A_{17}} \Psi_{22}) + A_{38} (\Psi_9 \Psi_{10}) + \\
&A_{39} e^{-A_{15} t} (\Psi_{23} \quad e^{-A_{15}} \Psi_{24}) + A_{40} (\Psi_1 \Psi_2) + A_{41} e^{-B_3 t} (\Psi_{11} \quad e^{-B_3} \Psi_{12}) + \\
&A_{42} (\Psi_5 \Psi_6) + A_{43} e^{-A_{17} t} (\Psi_{25} \quad e^{-A_{17}} \Psi_{26}) + A_{44} (\Psi_3 \Psi_4) + \\
&A_{45} e^{-B_3 t} (\Psi_7 \quad e^{-B_3} \Psi_8) \quad (35)
\end{aligned}$$

SKIN FRICTION

The co-efficient of skin friction at the plate is given by

$$\begin{aligned}
\tau &= \left. \frac{du}{dy} \right|_{y=0} = A_{46} (\Psi_{39} \Psi_{40}) \quad \Psi_{39} + A_{39} e^{-A_{15} t} (\Psi_{41} \quad e^{A_{15}} \Psi_{42}) + \\
&A_{47} (\Psi_{43} \Psi_{44}) + A_{48} e^{-B_3 t} (\Psi_{45} \quad e^{B_3} \Psi_{46}) + A_{43} e^{-A_{17} t} (\Psi_{47} \quad e^{A_{17}} \Psi_{48}) \\
&A_{38} (\Psi_{33} \Psi_{34}) - A_{39} e^{-A_{15} t} (\Psi_{49} \quad e^{A_{15}} \Psi_{50}) \quad A_{40} (\Psi_{29} \Psi_{30}) - \\
&A_{41} e^{-B_3 t} (\Psi_{35} \quad e^{B_3} \Psi_{36}) - A_{42} (\Psi_{31} \Psi_{32}) \quad A_{43} e^{-A_{17} t} (\Psi_{51} \quad e^{A_{17}} \Psi_{52}) \\
&A_{44} (\Psi_{27} \Psi_{28}) - A_{45} e^{-B_3 t} (\Psi_{37} \quad e^{B_3} \Psi_{38})
\end{aligned}$$

RATE OF HEAT TRANSFER

The co-efficient of the rate of heat transfer in terms of Nusslet number is given by

$$Nu = \left. \frac{d\theta}{dy} \right|_{y=0} = \Psi_{34} \Psi_{33}$$

RATE OF MASS TRANSFER

The co-efficient of the rate of mass transfer in terms of Sherwood number is given by

$$\text{Sh} = \left. \frac{\partial \phi}{\partial y} \right|_{y=0} = A_8(\Psi_{28} \Psi_{27}) + A_7(\Psi_{30} \Psi_{29}) + A_3 A_4 (\Psi_{34} \Psi_{33} + \Psi_{31} \Psi_{32}) + A_3 A_5 e^{-B_3 t} (\Psi_{37} \Psi_{35} + e^{B_3} (\Psi_{36} \Psi_{38})) (\Psi_{37} e^{B_3} \Psi_{38}) (\Psi_{35} e^{B_3} \Psi_{36})$$

RESULTS AND DISCUSSION

In order to get clear insight of the physical problem, numerical computations from the analytical Solutions for the representative temperature field, concentration field, velocity field, the co-efficient of skin friction, the rate of heat transfer at the plate in terms of Nusselt number and the rate of mass transfer in terms of Sherwood number have been carried out by assigning numerical values to the parameters involved in the problem, the normal coordinate y and time t . Throughout our investigation, the value of the Pr have been kept fixed at 0.71, both the values of Gr and Gm are fixed at 10 as the numerical computations are concerned. We recall that Pr=0.71 corresponds physically to air. The numerical results computed from the analytical solutions of the problem have been displayed in Figure 2-17. It is noticed Figures 2-7 correspond to the temperature distribution θ against y under the influence of Pr, Re and Q. Increasing Pr, Re or Q shows an opposing influence on θ indicating the fact that the fluid temperature falls steadily for large value of Pr, small viscosity (large Reynolds number) and for high radiation. It is seen that the magnitude of velocity goes up as the value of Da is increased.

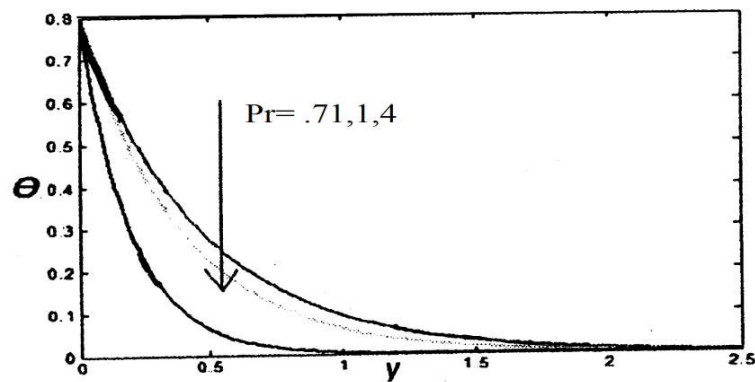


Fig. 2. Temperature versus y for $\text{Re}=1$, $Q=5$, $t=0.8$

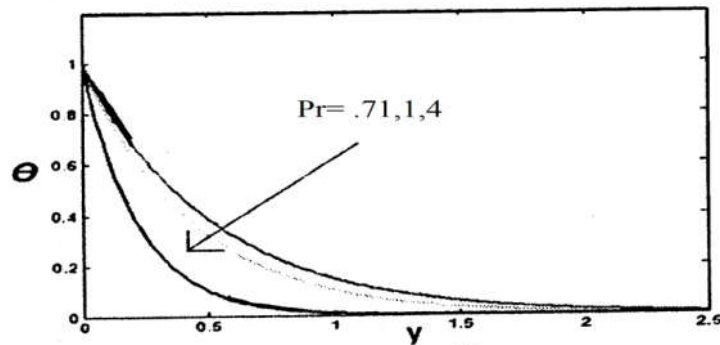


Fig. 3. Temperature versus y for $\text{Re}=1$, $Q=5$, $t=1.2$

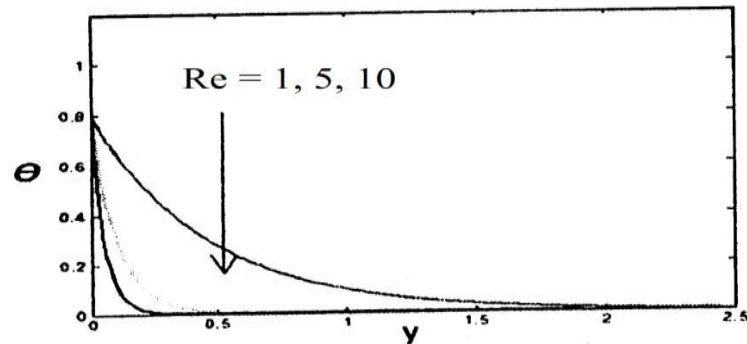


Fig. 4. Temperature versus y for $\text{Pr}=0.71$, $Q=5$, $t=0.8$

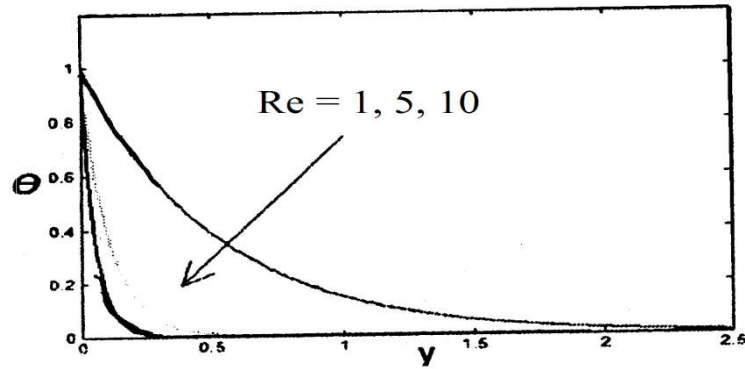


Fig.5. Temperature versus y for $Pr=0.71, Q=5, t=1.2$

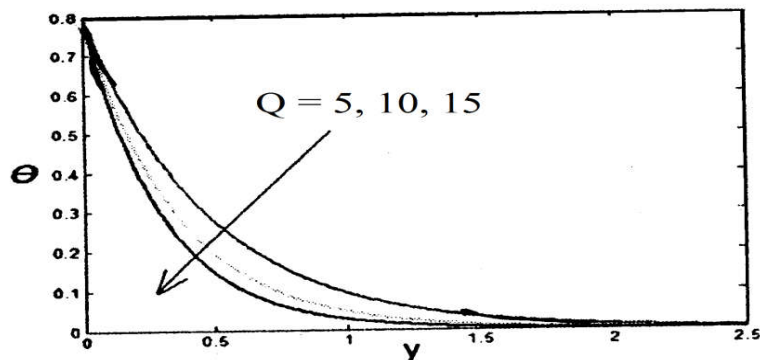


Fig. 6. Temperature versus y for $Pr=0.71, Re=1, t=0.8$

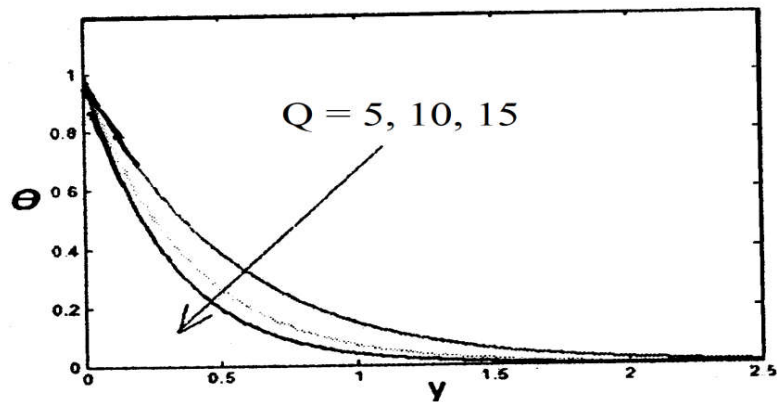


Fig. 7. Temperature versus y for $Pr=0.71, Re=1, t=1.2$

We also observe from these figures that it is immaterial whether the plate temperature is uniform or as the effect of Pr , Q or Re on θ is concerned. It is further noticed from these figures that as expected the fluid temperature asymptotically falls from its maximum value at $y=0$ to its minimum value at $y \rightarrow \infty$. Figures 8 and 9 show that the concentration level of the fluid falls as Sc increases whether the plate is of ramped temperature or isothermal temperature indicating the fact that mass diffusion raises the Concentration level of the fluid. We also notice that in case of isothermal plate (figure 9) that the Concentration field first increases in a thin layer adjacent to the plate and there after it decreases asymptotically to approach its minimum value far away from the plate. When the strength of applied magnetic field is increased the primary skin friction increases and falls due to Hall Effect. In case of ramped temperature (Figure 8), it is seen that the concentration distribution falls asymptotically from its maximum value 1 at $y=0$ to its minimum value at $\theta=0$ as $y \rightarrow \infty$.

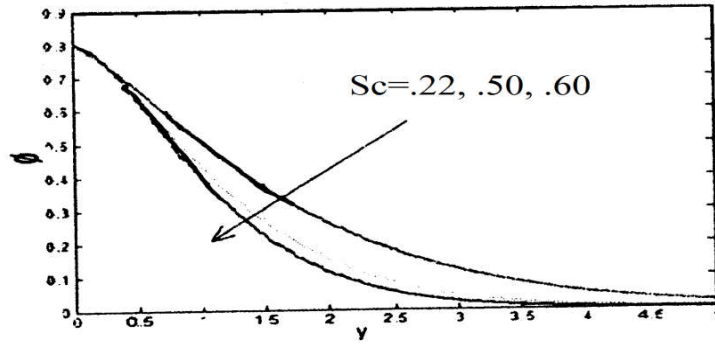


Fig. 8. Concentration versus y for $Re=1, Q=5, Sr=1, t=8$

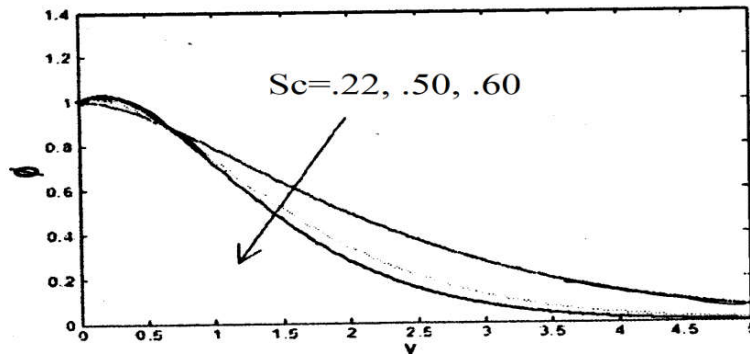


Fig. 9. Concentration versus y for $Re=1, Q=5, Sr=1, t=1.2$

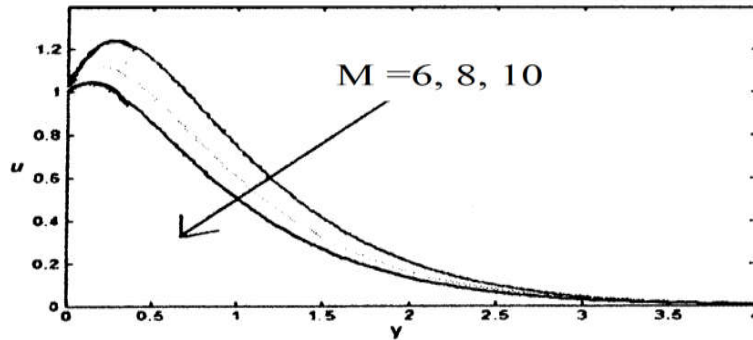


Fig. 10. Velocity versus y for $Gr=10, Gm=10, Pr=0.71, Sc=0.6, Re=1, Q=5, Sr=1, t=0.8$

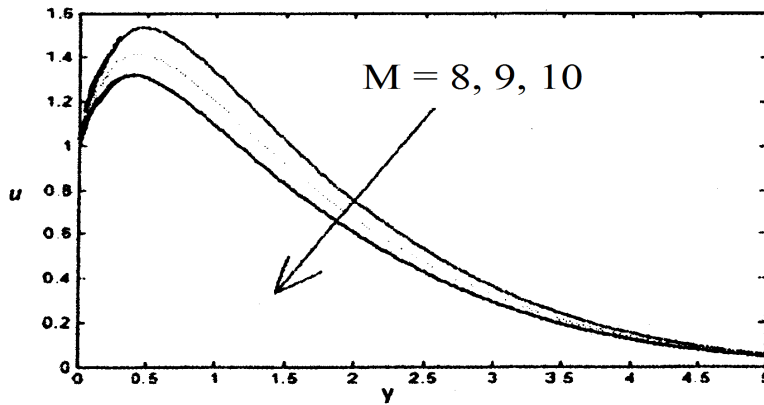


Fig. 11. Velocity versus y for $Gr=10, Gm=10, Pr=0.71, Sc=0.6, Re=1, Q=5, Sr=1, t=1.2$

Figures 10 and 11 present how the fluid velocity is affected by applied magnetic field for the cases $0 < t \leq 1$ and $t > 1$ respectively. These two figures uniquely show that an increase in the Hartmann number results in a steady decrease in the fluid velocity thereby reducing the thickness of the velocity boundary layer. These two figures further reveal that the fluid velocity first increases in a thin layer adjacent to the plate and there after it decreases asymptotically as we move away from the plate indicating the fact that the buoyancy force has a significant effect on the flow near the plate and its effect is mollified in the free stream.

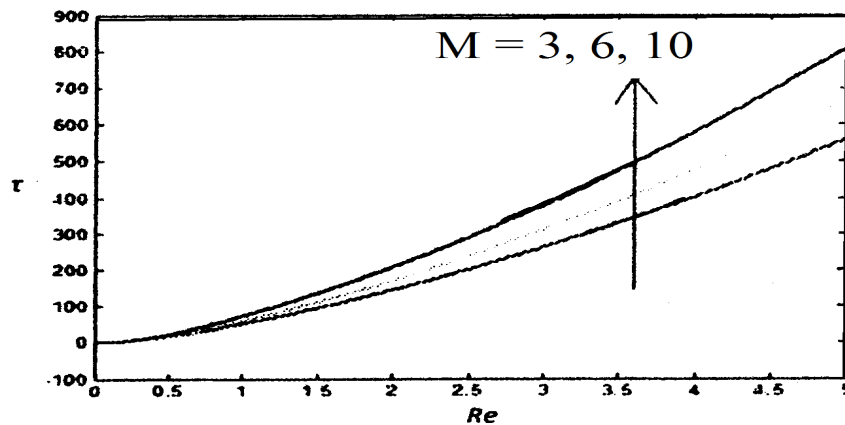


Fig. 12. Coefficient of skin friction versus Re for $Gr=10$, $Gm=10$, $Pr=0.71$, $Sc=.6$, $Q=5$, $Sr=1$, $t=0.8$

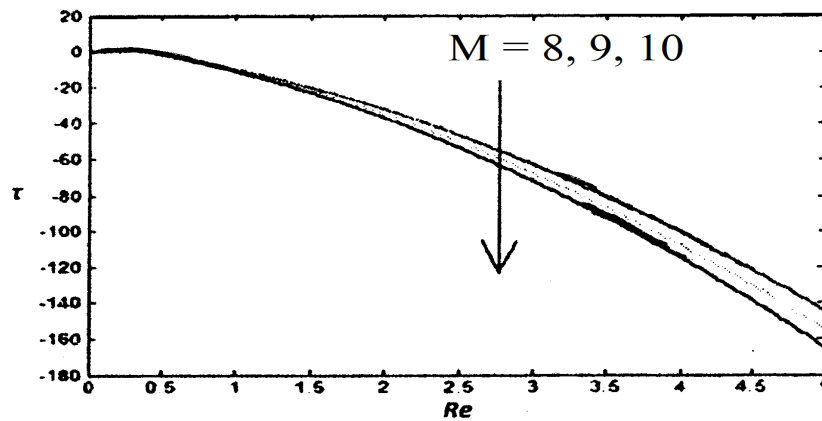


Fig. 13. Coefficient of skin friction versus Re for $Gr=10$, $Gm=10$, $Pr=0.71$, $Sc=.6$, $Q=5$, $Sr=1$, $t=1.2$

Figures 12 and 13 demonstrate the effect of the Hartmann number M and Reynolds number Re on the skin friction at the plate in the direction of the plate velocity. Both the figures indicate that the magnitude of shear stress at the plate is considerably increased with the increase in M and Re . The effects of the radiation parameter Q and the Reynolds number Re on the co-efficient of the rate of heat transfer in terms of the Nusselt number Nu have been displayed in figures 14 and 15. These figures predict that Nu is constantly increased for increasing Q or Re . This simulates that the high radiation or small viscosity leads the substantial rise in the heat transfer rate. The effect of Schmidt number on the rate mass transfer from the plate to the fluid for both ramped temperature and isothermal plate has been visualized in figures 16 and 17. Both the figures exhibit that rate of mass transfer coefficients gets enhanced for increasing Sc as well as Re . It is recalled the that Sc increases means mass diffusion decreases i.e. the rate of mass flux falls as the mass diffusivity increases for both the cases(ramped and isothermal).

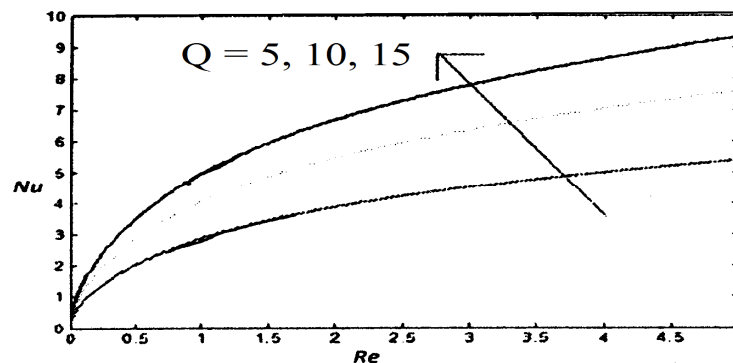
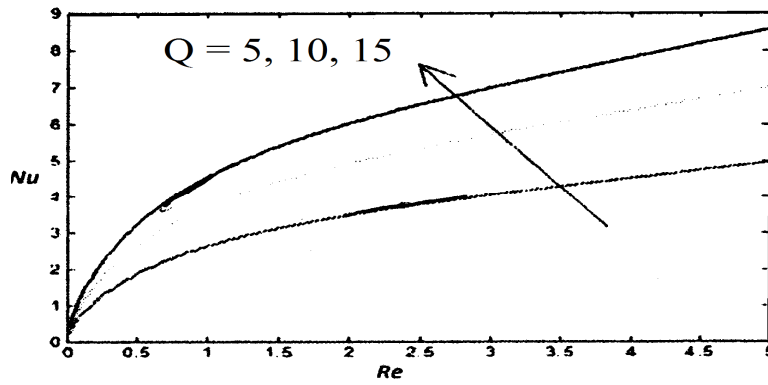
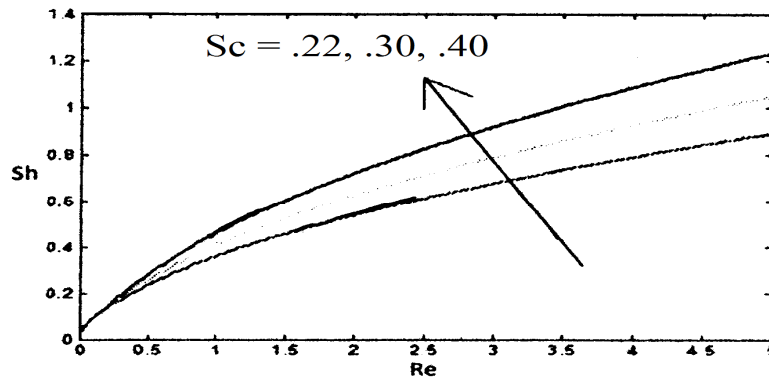
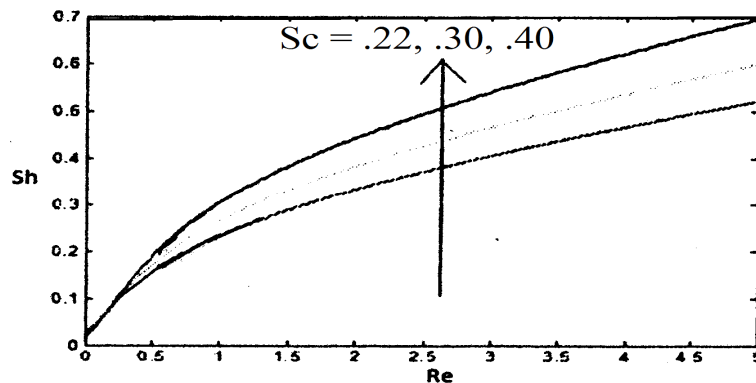


Fig. 14. Nusselt number versus Re for $Pr=0.71$, $t=0.8$

Fig. 15. Nusselt number versus Re for $Pr=0.71$, $t=1.2$ Fig. 16. Sherwood number versus Re for $Pr=0.71$, $Q=5$, $Sr=1$, $t=0.8$ Fig. 17. Sherwood number versus Re for $Pr=0.71$, $Q=5$, $Sr=1$, $t=1.2$

The same figures also establish the fact that the Sherwood number Sh increases comprehensively for increasing Reynolds number. In other words the rate of mass transfer rises for low viscosity. Further it is noticed that the effect of Schmidt number on Sherwood number is almost nil for small values of Re (i.e. for high viscosity).

Conclusion

- The imposition of the transverse magnetic field retards the flow. As a consequence of this, the growth of thickness of the velocity boundary layer is prevented to some extent which in turn stabilizes the flow.
- Magnitude of velocity increases for higher value of Darcy number.
- Large value of prandtl number, small viscosity or high radiation causes the fluid temperature to fall and thereby reduces the thickness of the thermal boundary layer.
- Magnitude of shear stress at the plate is considerably increased due to application of transverse magnetic field.
- High radiation or small viscosity leads the substantial rise in the heat transfer rate.
- Rate of mass flux increases with the increase the values of Schmidt number.
- Skin friction and Nusselt number are enhanced due to the increase of the radiation parameter.

NOMENCLATURE

\vec{B} is the Magnetic induction vector,
 B_0 is the strength of the applied magnetic field,
 C_p is the specific heat at constant pressure,
 \bar{C} is the species concentration,
 D_M is the coefficient of chemical molecular diffusivity,
 D_T is the coefficient of chemical thermal diffusivity,
 \vec{E} is the electric field,
 $e_{b\lambda}$ is the Plank function,
 \bar{f} is the Laplace transform of f ,
 g is the gravitational acceleration vector,
 Gr is the Grashof number for Heat transfer,
 Gm is the Grashof number for Mass transfer,
 J is the current density vector,
 K is the thermal conductivity,
 M is the Hartmann number,
 Pr is the Prandtl number,
 P is the pressure,
 q is the fluid velocity vector,
 Q is the radiation parameter,
 q_r is the radiative flux,
 Re is the Reynolds number,
 t^1 is the time,
 t_0 is the characteristic time,
 t_w^1 is the reference temperature,
 t_∞^1 is the temperature far away from the plate,
 T is the non dimensional time,
 T^1 is the fluid temperature,
 U_0 is the plate velocity,
 ρ is the fluid density,
 μ is the co-efficient of viscosity,
 σ is the electrical conductivity,
 φ is the viscous dissipation of energy per unit volume,
 ρ_∞ is the density far away from the plate,
 δn^1 is an element of the outward normal,
 β is the volumetric co-efficient of thermal expansion,
 λ is the wave length,
 θ is the non-dimensional temperature,
 \bar{c} is the non dimensional concentration,

APPENDIX

$$\Psi_1 = f(PrRe, Q Re, y, t), \Psi_2 = f(PrRe, Q Re, y, t - 1) H(t - 1), \Psi_3 = f(ScRe, y, t),$$

$$\Psi_4 = f(ScRe, y, t - 1) H(t - 1), \Psi_5 = \chi(ScRe, y, t), \Psi_6 = \chi(ScRe, y, t - 1) H(t - 1),$$

$$\Psi_7 = \chi(ScRe, B_3, y, t), \Psi_8 = \chi(ScRe, B_3, y, t - 1) H(t - 1), \Psi_9 = \chi(PrRe, Q Re, y, t),$$

$$\Psi_{10} = \chi(PrRe, Q Re, y, t - 1) H(t - 1), \Psi_{11} = \chi(PrRe, Q Re, B_3, y, t),$$

$$\Psi_{12} = \chi(PrRe, Q Re, B_3, y, t - 1) H(t - 1), \Psi_{13} = \chi(Re, MRe, y, t),$$

$$\Psi_{14} = \chi(Re, MRe, y, t - 1) H(t - 1), \Psi_{15} = \chi(Re, MRe, A_{15}, y, t),$$

$$\Psi_{16} = \chi(Re, MRe, A_{15}, y, t - 1) H(t - 1), \Psi_{17} = f(Re, MRe, y, t),$$

$$\Psi_{18} = f(Re, MRe, y, t - 1) H(t - 1), \Psi_{19} = \chi(Re, \xi, y, t),$$

$$\Psi_{20} = \chi(Re, MRe, B_3, y, t - 1) H(t - 1), \Psi_{21} = \chi(Re, MRe, A_{17}, y, t),$$

$$\Psi_{22} = \chi(\text{Re}, M \text{Re}, A_{17}, y, t) H(t-1), \quad \Psi_{23} = \chi(\text{PrRe}, Q \text{Re}, A_{15}, y, t),$$

$$\Psi_{24} = \chi(\text{PrRe}, Q \text{Re}, A_{15}, y, t) H(t-1), \quad \Psi_{25} = \chi(\text{ScRe}, A_{17}, y, t),$$

$$\Psi_{26} = \chi(\text{ScRe}, A_{17}, y, t) H(t-1),$$

$$A_1 = \text{ScSrPrRe}, A_2 = \text{PrRe} - \text{ScRe}, A_3 = \frac{A_1}{A_2}, B_1 = Q \text{Re}, \quad B_2 = \text{PrRe}^2 Q, \quad B_3 = \frac{B_2}{A_2},$$

$$A_4 = \frac{B_1 - B_3}{B_3}, A_5 = A_4, A_6 = \frac{B_1 - B_3}{B_3}, A_7 = A_3 + A_3 A_6, A_8 = 1, A_7,$$

$$A_9 = \text{PrRe} - \text{Re}, A_{10} = \text{PrRe}^2 Q - M \text{Re}^2, A_{11} = \text{Gr} + \text{Gm} A_7, A_{12} = \text{ScRe} - \text{Re},$$

$$A_{13} = M \text{Re}^2, A_{14} = \frac{\text{Re}^2}{A_9}, A_{15} = \frac{A_{10}}{A_9}, A_{16} = \frac{G M \text{Re}^2}{A_{12}}, A_{17} = \frac{A_{13}}{A_{12}},$$

$$A_{18} = A_{14} A_{11}, A_{19} = \frac{1}{A_{15}^2}, A_{20} = \frac{1}{A_{15}}, A_{21} = \frac{1}{A_{15}^2}, A_{22} = \text{Gm} A_3 A_{14} A_4,$$

$$A_{23} = \frac{1}{A_{15}}, A_{24} = \frac{1}{A_{15}}, A_{25} = \text{Gm} A_3 A_{14} A_5, A_{26} = \frac{1}{B_3 - A_{15}}, A_{27} = \frac{1}{A_{15} - B_3}, A_{28} = A_{16} A_8, A_{29} = \frac{1}{A_{17}^2}, A_{30} = \frac{1}{A_{17}^2}$$

$$, A_{31} = \frac{1}{A_{17}}, A_{32} = A_3 A_4 A_{16}, A_{33} = \frac{1}{A_{17}}, A_{34} = \frac{1}{A_{17}}, A_{35} = A_3 A_5 A_{16}, A_{36} = \frac{1}{B_3 - A_{17}}, A_{37} = \frac{1}{A_{17} - B_3},$$

$$A_{38} = A_{18} A_{19} + A_{22} A_{24}, A_{39} = A_{25} A_{26} + A_{18} A_{20} + A_{22} A_{23}, A_{40} = A_{18} A_{21},$$

$$A_{41} = A_{25} A_{27}, A_{42} = A_{28} A_{29}, A_{32} A_{34}, A_{43} = A_{28} A_{30}, A_{32} A_{33}, A_{35} A_{36},$$

$$A_{44} = A_{28} A_{31}, A_{45} = A_{35} A_{37}, A_{46} = A_{38} + A_{42}, A_{47} = A_{40} + A_{44}, A_{48} = A_{41} + A_{45},$$

$$\Psi_{27} = \varphi(\text{ScRe}, t), \Psi_{28} = \varphi(\text{ScRe}, t) H(t-1), \quad \Psi_{29} = \zeta(\text{PrRe}, Q \text{Re}, t),$$

$$\Psi_{30} = \zeta(\text{PrRe}, Q \text{Re}, t) H(t-1), \Psi_{31} = \Omega(\text{Sc}, \text{Re}, t), \Psi_{32} = \Omega(\text{Sc}, \text{Re}, t) H(t-1),$$

$$\Psi_{33} = \Phi(\text{PrRe}, Q \text{Re}, t), \Psi_{34} = \Phi(\text{PrRe}, Q \text{Re}, t) H(t-1),$$

$$\Psi_{35} = \Phi(\text{PrRe}, Q \text{Re}, B_3, t), \Psi_{36} = \Phi(\text{PrRe}, Q \text{Re}, B_3, t) H(t-1),$$

$$\Psi_{37} = \Phi(\text{ScRe}, B_3, t), \Psi_{38} = \Phi(\text{ScRe}, B_3, t) H(t-1), \Psi_{39} = \Phi(\text{Re}, M \text{Re}, t),$$

$$\Psi_{40} = \Phi(\text{Re}, M \text{Re}, t) H(t-1), \quad \Psi_{41} = \Phi(\text{Re}, M \text{Re}, A_{15}, t),$$

$$\Psi_{42} = \Phi(\text{Re}, M \text{Re}, A_{15}, t) H(t-1), \quad \Psi_{43} = \zeta(\text{Re}, M \text{Re}, t),$$

$$\Psi_{44} = \zeta(\text{Re}, M \text{Re}, t) H(t-1), \quad \Psi_{45} = \Phi(\text{Re}, M \text{Re}, B_3, t),$$

$$\Psi_{46} = \Phi(\text{Re}, M \text{Re}, B_3, t) H(t-1), \quad \Psi_{47} = \Phi(\text{Re}, M \text{Re}, A_{17}, t),$$

$$\Psi_{48} = \Phi(\text{Re}, M \text{Re}, A_{17}, t) H(t-1), \quad \Psi_{49} = \Phi(\text{PrRe}, Q \text{Re}, A_{15}, t),$$

$$\Psi_{50} = \Phi(\text{PrRe}, Q \text{Re}, A_{15}, t) H(t-1), \quad \Psi_{51} = \Phi(\text{ScRe}, A_{17}, t),$$

$$\Psi_{52} = \Phi(\text{ScRe}, A_{17}, t) H(t-1),$$

$$\chi(\xi, \eta, y, t) = \frac{1}{2} \left[e^{\sqrt{\xi \eta y}} \text{erfc} \left(\frac{y \sqrt{\xi}}{2 \sqrt{t}} + \sqrt{\eta t} \right) + e^{-\sqrt{\xi \eta y}} \text{erfc} \left(\frac{y \sqrt{\xi}}{2 \sqrt{t}} - \sqrt{\eta t} \right) \right]$$

$$f(\xi, \eta, y, t) = \left(\frac{t}{2} + \frac{y \sqrt{\xi}}{4 \sqrt{\eta}} \right) e^{\sqrt{\xi \eta y}} \text{erfc} \left(\frac{y \sqrt{\xi}}{2 \sqrt{t}} + \sqrt{\eta t} \right) + \left(\frac{t}{2} - \frac{y \sqrt{\xi}}{4 \sqrt{\eta}} \right) e^{-\sqrt{\xi \eta y}} \text{erfc} \left(\frac{y \sqrt{\xi}}{2 \sqrt{t}} - \sqrt{\eta t} \right)$$

$$\Omega(\xi, \eta, t) = \sqrt{\frac{\xi \eta}{t \pi}}, \quad \varphi(\xi, t) = 2 \sqrt{\frac{t}{\pi}}, \quad \Phi(\xi, \eta, t) = \frac{\sqrt{\xi}}{\sqrt{\pi t}} e^{-\eta t} - \text{erf}(\sqrt{\eta t}) \sqrt{\xi} \sqrt{\eta}$$

$$\zeta(\xi, \eta, t) = \sqrt{\frac{\xi t}{\pi}} e^{-\eta t} - \left(t \sqrt{\xi} + \frac{\sqrt{\xi t}}{2 \sqrt{\eta}} \right) \text{erf}(\sqrt{\eta t}), \quad H(t-1) = \begin{cases} 0, & t < 1 \\ 1, & t > 1 \end{cases}$$

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