Available online at http://www.journalijdr.com

International Journal of DEVELOPMENT RESEARCH

ISSN: 2230-9926

International Journal of Development Research
Vol. 06, Issue, 08, pp.9264-9270, August, 2016

Full Length Research Article

ON THE RELATIONSHIP BETWEEN ELZAKI TRANSFORM AND NEW INTEGRAL TRANSFORM "ZZ TRANSFORM"

[1,2]Abdelbagy A. Alshikh and [1,2]Mohand M. Abdelrahim Mahgob

[1]Department of Mathematics, Faculty of Sciences and Arts-Almikwah-Albaha University- Saudi Arabia
[2]Department of Mathematics, Faculty of Education-Alzaeim Alazhari University-Sudan

ARTICLE INFO

Article History:

Received 15th May, 2016
Received in revised form
29th June, 2016
Accepted 17th July, 2016
Published online 31st August, 2016

Key Words:

Elzaki Transform,
ZZ Transform,
Differential equations.

ABSTRACT

In this paper we discusses some relationship between Elzaki transform and the newintegral transform called ZZ transform, we solve first and second order ordinarydifferential equations with constant and non-constant coefficients, using both transforms,and showing ZZ transform is closely connected with Elzaki transform.

INTRODUCTION

There are several integral transforms (Miles, 1971) like, Laplace Transform, Fourier Transform, Sumudu Transform, ElzakiTransform, Natural Transform and Aboodh Transform, tocrack the DEs and IEs. Of these the most widely used transform is Laplace Transform. Elzaki transform (Tarig and Elzaki , 2011; Tarig M. Elzaki and Salih M. Elzaki, 2011; Tarig M. Elzaki and Salih M. Elzaki, 2011; Tarig M. Elzaki, AdemKilicman, Hassan Eltayeb, 2010 and Tarig M. Elzaki 2009), which is a modified general Laplace and Sumudutrans forms, has been shown to solve effectively, easily and accurately a large classof linear differential equations. Elzaki transform was successfully applied to integralequations, partial differential equations , ordinary differential equations withvariable coefficients, and system of all these equations. New integraltransform, named as ZZ Transformation (Zain UlAbadin Zafar, 2016; ZainUlAbadinZafar, 2012; Zafar, 2008 and ZainUlAbadin Zafar, 2013), intrpduce by ZainUlAbadin Zafar (2016), ZZ transform was successfully applied to integral equations, ordinary differential equations. the main objective is to introduce a comparative study to solve differential equations by using Elzaki transform and ZZ transform. The plane of the paper is as follows: In section 2, we introduce the basic idea of Elzaki transform, then, ZZ Transform in 3, Application in 4 and conclusion in 5, respectively.

Definitions and Standard ruselts

2. ElzakiTrans form
2.1 Definition :

Elzaki Transform (2). Given a function $f(t)$defined for all $t \geq 0$, as follow:

***Corresponding author: [1,2]Abdelbagy A. Alshikh**
[1]*Department of Mathematics, Faculty of Sciences and Arts-Almikwah-Albaha University- Saudi Arabia*
[2]*Department of Mathematics, Faculty of Education-Alzaeim Alazhari University-Sudan*

$E[f(t),v] = T(v) = vP\int_0^t f(t)e^{-\frac{t}{v}}dt \quad , \quad v \in (k_1, k_2)$ (1)

for all values of s, for which the improper integral converges

2.2 Elzaki transform of some functions :

$E(1) = v^2, \quad E(t^n) = n!\,v^{n+2}, \quad E(e^{at}) = \frac{v^2}{1-av}$

$E(\sin(at)) = \frac{av^3}{1+a^2v^2} \quad , \; E(\cos(at)) = \frac{v^2}{1+a^2v^2}.$

2.2 Elzaki transform of derivatives :

(i) $E[f'(t)] = \frac{T(u)}{u} - uf(0)$

(ii) $E[f''(t)] = \frac{T(u)}{u^2} - f(0) - uf'(0)$

3: The ZZ Transform: 3.1 Definition:

Let (t) be a function defined for all $t \geq 0$. The ZZ transform of $f(t)$is the function $Z(u,s)$ defined by

$Z(u,s) = H\{f(t)\} = s\int_0^\infty f(ut)e^{-st}dt$ (2)

provided the integral on the right side exists. The unique function $f(t)$in (2) is called the inverse transform of $Z(u,s)$is indicated by

$f(t) = H^{-1}\{Z(u,s)\}$

Equation (2) can be written as

$H\{f(t)\} = \frac{s}{u}\int_0^\infty f(t)e^{-\frac{s}{u}t}dt$ (3)

3.2 ZZ transform of some functions :

$H\{1\} = 1 \quad , \quad H\{t^n\} = n!\frac{u^n}{s^n}, \quad H\{e^{at}\} = \frac{s}{s-ua}$

$E(\sin(at)) = \frac{aus}{s^2+a^2u^2} \quad , \; E(\cos(at)) = \frac{s^2}{s^2+a^2u^2}.$

3.3 ZZ transform of derivatives :

1)) let $H\{f(t)\} = Z(u,s)$ then

$H\{tf^{(n)}(t)\} = \frac{s^n}{u^n}Z(u,s) \quad \sum_{k=0}^{n-1}\frac{s^{n-k}}{u^{n-k}}f^{(k)}(0)$

2)) (i) $H\{tf(t)\} = \frac{u^2}{s}\frac{d}{du}\big(Z(u,s)\big) + \frac{u}{s}Z(u,s)$

(ii)) $H\{tf'(t)\} = \frac{u^2}{s}\frac{d}{du}\left(\frac{s}{u}Z(u,s)\right) + Z(u,s)$

(iii) $H\{tf''(t)\} = s\frac{d}{du}\big(Z(u,s)\big) \quad \frac{s}{u}Z(u,s) + \frac{s}{u}f(0)$

4 Application :

Example 4.1consider the first order differential equation

$\frac{dy}{dx} + y = 0$ (4)

With the initial condition; $, y(0) = 1$ (5)

Solution:

1:Applying the Elzaki transform of both sides of Eq. (4),

$$E\left\{\frac{dy}{dx}\right\} + E\{y\} = E\{0\} \tag{6}$$

Using the differential property of Elzaki transform Eq.(6) can be written as:

$$\frac{1}{v}E(y) \quad vy(0) + E(y) = 0 \tag{7}$$

Using initial condition (5), Eq. (7) can be written as:

$$E(y) = \frac{v^2}{1+v} \tag{8}$$

The inverse Elzaki transform of this equation is simply obtained as

$$y(x) = e^{-x} \tag{9}$$

Where $E(y)$Is the Elzaki transform of the function $y\,(x\,)$

2: Applying the ZZ transform of both sides of Eq. (4),

$$H\left\{\frac{dy}{dx}\right\} + H\{y\} = H\{0\},\ \text{so} \tag{10}$$

Using the differential property of ZZ transform Eq.(6) can be written as:

$$\frac{s}{u}z(u,s) \quad \frac{s}{u}y(0) + z(u,s) = 0 \tag{11}$$

Using initial condition (5), Eq. (11) can be written as

$$z(u,s)\left[\frac{s}{u}+1\right] = \frac{s}{u} \quad , \quad z(u,s) = \frac{s}{s+u} \tag{12}$$

The inverse ZZ transform of this equation is simply obtained as

$$y(x) = e^{-x} \tag{13}$$

Where H is the ZZ transform of the function $y\,(x\,)$

Example 4.2 Let us consider the second-order differential equation

$$y'' + y = 0 \quad , \tag{14}$$

With the initial condition; $, y(0) = y'(0) = 1$ (15)

Solution:

1: Applying the Elzaki transform of both sides of Eq. (14),

$\text{E}\{\,y''\} + \text{E}\{y\} = \text{E}\{0\}$ (16)

Using the differential property of Elzaki transform Eq.(16) can be written as

$$\frac{1}{v^2}E(y) \quad 1 + E(y) \quad v = 0 \tag{17}$$

Using initial condition (15), Eq. (17) can be written as

$$E(y) = \frac{v^2}{v^2+1} + \frac{v^3}{v^2+1} \tag{18}$$

The inverse Elzaki transform of this equation is simply obtained as

$$y(x) = \cos x + \sin x \tag{19}$$

2:Applying the ZZ transform of both sides of Eq. (14),

$\mathrm{H}\{ y''\} + \mathrm{H}\{y\} = \mathrm{E}\{0\}$, so (20)

Using the differential property of ZZ transform Eq.(20) can be written as:

$$\frac{s^2}{u^2}Z(u,s) \quad \frac{s^2}{u^2}y(0) \quad \frac{s}{u}y`(0) + Z(u,s) = 0 \tag{21}$$

Using initial condition (15), Eq. (21) can be written as

$$\frac{s^2}{u^2}Z(u,s) \quad \frac{s^2}{u^2} \quad \frac{s}{u} + Z(u,s) = 0, \text{ therefore}$$

$$Z(u,s) = \frac{s^2}{s^2+u^2} + \frac{su}{s^2+u^2} \tag{22}$$

The inverse ZZ transform of this equation is simply obtained as

$$y(x) = \cos x + \sin x \tag{23}$$

Example 4.3 Consider the second-order differential equation

$$y'' \quad 3y' + 2y = 0 \tag{24}$$

With the initial condition, $y(0) = 1$, $y'(0) = 4$ (25)

Solution:

1: Applying the Elzaki transform of both sides of Eq. (24),

$$\mathrm{E}\{ y''\} \quad \mathrm{E}\{3y'\} + \mathrm{E}\{2y\} = \mathrm{E}\{0\} \tag{26}$$

Using the differential property of Elzaki transform Eq.(26) can be written as

$$E(y) = \frac{v^2(v+1)}{(2v \quad 1)(v \quad 1)} = v^2\left[\frac{2}{v \quad 1} \quad \frac{3}{2v \quad 1}\right]$$

$$E(y) = \frac{-2v^2}{1-v} + \frac{3v^2}{1-2v} \tag{27}$$

Then take the inverse of ELzaki transform we get

y(x) = $-2e^{t} + 3e^{2t}$. (28)

3:Applying the ZZ transform of both sides of Eq. (24),:

$\mathrm{H}\{ y''\} \quad \mathrm{H}\{3y'\} + \mathrm{H}\{2y\} = \mathrm{E}\{0\}$, so (29)

Using the differential property of ZZ transform Eq.(29) can be written as:

$$\frac{s^2}{u^2}Z(u,s) \quad \frac{s^2}{u^2}f(0) \quad \frac{s}{u}f'(0) \quad 3\left(\frac{s}{u}z(u,s) \quad \frac{s}{u}f(0)\right) + 2Z(u,s) = 0 \tag{30}$$

Using initial condition (25), Eq. (30) can be written as

$$\frac{s^2}{u^2}Z(u,s) \quad \frac{s^2}{u^2} \quad 4\frac{s}{u} \quad 3\frac{s}{u}Z(u,s) \quad 3\frac{s}{u} + 2Z(u,s) = 0$$

$$Z(u,s) = \frac{su+s^2}{(s-u)(s-2u)} \tag{31}$$

Solve equation (31) Then take the inverse of ZZ transform we get

A(x) = $-2e^{t} + 3e^{2t}$ (32)

Example4.4

Consider the initial value problem

$$y''(t) + 2y'(t) + 5y(t) = e^{-t} sin\ t \tag{33}$$

With the initial conditions

$$y(0) = 0\ ,\ y'(0) = 0 \tag{34}$$

Solution:

1: Applying the Elzaki transform of both sides of Eq. (33),

$$\mathrm{E}\{y''(t)\} + \mathrm{E}\{2y'(t)\} + \mathrm{E}\{5y(t)\} = \mathrm{E}\{e^{-t} sin\ t\} \tag{35}$$

Using the differential property of Elzaki transform Eq.(35) can be written as

$$\frac{T(v)}{v^2} \quad f(0) \quad vf'(0) + 2\left(\frac{T(v)}{v} \quad vf \quad (0)\right) + 5T(v) = \frac{v}{\left(1+\frac{1}{v}\right)^2 + 1} \tag{36}$$

Now applying the initial condition to obtain

$$\frac{T(v)}{v^2} \quad v + 2\frac{T(v)}{v} + 5T(v) = \frac{v}{\left(1+\frac{1}{v}\right)^2 + 1}$$

$$T(v) = \frac{v}{\left(\left(1+\frac{1}{v}\right)^2 + 1\right)\left(\left(1+\frac{1}{v}\right)^2 + 4\right)} + \frac{v}{\left(\left(1+\frac{1}{v}\right)^2 + 4\right)}$$

$$T(v) = \frac{1}{3}\left(\frac{v}{\left(\left(1+\frac{1}{v}\right)^2+1\right)} \quad \frac{v}{\left(\left(1+\frac{1}{v}\right)^2+4\right)}\right) + \frac{v}{\left(\left(1+\frac{1}{v}\right)^2+4\right)} \tag{37}$$

Now applying the inverse Elzaki transform, we get

$$y(t) = \frac{1}{3}e^{-t}\sin t + \frac{1}{3}e^{-t}\sin 2t$$

$$y(t) = \frac{1}{3}e^{-t}(\sin t + \sin 2t). \tag{38}$$

2: Applying the ZZ transform to both sides of (33) we have

$$H\{\ y''(t)\} + 2H\{y'(t)\} + 5H\{y(t)\} = H\{e^{-t} sin\ t\} \tag{39}$$

Using the differential property of ZZ transform Eq.(29) can be written as:

$$\frac{s^2}{u^2}Z(u,s) \quad \frac{s^2}{u^2}y(0) \quad \frac{s}{u}y'(0) + 2\left(Z(u,s) \quad \frac{s}{u}y(0)\right) + 5Z(u,s) = \frac{\frac{s}{u}}{\left(\frac{s}{u}+1\right)^2+1} \tag{40}$$

Now applying the initial condition to obtain

$$\left(\frac{s^2}{u^2} + 2 + 5\right)Z(u,s) = \frac{s}{u} + \frac{\frac{s}{u}}{\left(\frac{s}{u}+1\right)^2 + 1}$$

$$Z(u,s) = \frac{1}{3}\left(\frac{\frac{s}{u}}{\left(\left(\frac{s}{u}+1\right)^2+1\right)} \quad \frac{\frac{s}{u}}{\left(\left(\frac{s}{u}+1\right)^2+4\right)}\right) + \frac{\frac{s}{u}}{\left(\left(\frac{s}{u}+1\right)^2+4\right)} \tag{41}$$

Now applying the inverse ZZ transform, we get

$$\mathrm{y(t)} = \frac{1}{3}\mathrm{e}^{-\mathrm{t}}\sin \mathrm{t} + \frac{1}{3}\mathrm{e}^{-\mathrm{t}}\sin 2\mathrm{t}$$

$$\mathrm{y(t)} = \frac{1}{3}\mathrm{e}^{-\mathrm{t}}(\sin \mathrm{t} + \sin 2\mathrm{t}) \tag{42}$$

Example 4.5

Consider the initial value problem

$$ty''(t) \quad ty'(t) + y(t) = 2 \tag{43}$$

With the initial conditions

$$y(0) = 2\ ,\ y'(0) = \quad 1 \tag{44}$$

Soluton:

1. Applying the Elzaki transform to both sides of (43) we have

$$\mathrm{E}\{ty''(t)\} \quad \mathrm{E}\{ty'(t)\} + \mathrm{E}\{y(t)\} = \mathrm{E}\{2\} \tag{45}$$

Using the differential property of Elzaki transform Eq.(45) can be written as

$$v^2\frac{d}{dv}\left[\frac{T(v)}{v^2} \quad f(0) \quad vf'(0)\right] \quad v \quad \left[\frac{T(v)}{v^2} \quad f(0) \quad vf'(0)\right]$$

$$v^2\frac{d}{dv}\left[\frac{T(v)}{v} \quad vf \quad (0)\right] + v \quad \left[\frac{T(v)}{v} \quad vf \quad (0)\right] + T(v) = 2v^2 \tag{46}$$

$$E[y(t)] = T(v)$$

Now applying the initial condition to obtain

$$(1 \quad v)T'(v) + 3\left(1 \quad \frac{1}{v}\right)T(v) = \quad v^2 \quad 2v^2$$

Or

$$T'(v) \quad \frac{3}{v}T(v) = \quad \frac{v^2-2v^2}{1-v}. \tag{47}$$

Equation (47) is a linear differential equation , which has solution in the form

$T(v) = cv^3 + 2v^2$(c = constant)

Now applying the inverse Elzaki transform, we get

$$y(t) = ct + 2 \tag{48}$$

2: Applying the ZZ transform to both sides of (43) we have

$$H\{\ y''(t)\} \quad H\{y'(t)\} + H\{y(t)\} = H\{2\} \tag{49}$$

Using the differential property of ZZ transform Eq.(49) can be written as

$$s\frac{d}{du}Z(u,s) \quad \frac{s}{u}Z(u,s) + \frac{s}{u}f(0) + \frac{u}{s}\frac{d}{du}\frac{s}{u}Z(u,s) + Z(u,s) + Z(u,s) = 2$$

Now applying the initial condition to obtain

$$sZ'(u,s) \quad \frac{s}{u}Z(u,s) + \frac{2s}{u} \quad \frac{u^2}{s}\left(\frac{s}{u}Z'(u,s)\right) \quad \frac{s}{u^2}Z(u,s) = 2$$

$$Z'(u,s) \quad \frac{1}{u}Z(u,s) = \quad \frac{2}{u} \tag{50}$$

Equation (50) is a linear differential equation , which has solution in the form

$$Z(u,s) = 2 + cu$$

Now applying the inverse ZZ transform, we get

$$y(t) = 2 + ct \tag{51}$$

4) Conclusions

The main goal of this paper is to conduct a comparative study between ELzaki Transform and new integrals "ZZ Transform" . The tow methods are powerful and efficient. Elzaki transform and ZZ transform is a convenient tool for solving differential equations in the time domain.

REFERENCES

Miles, J.W. 1971. Integral Transforms in Applied Mathematics, Cambridge: CambridgeUniversity Press.

Tarig M. Elzaki and Salih M. Elzaki, 2011. Applications of New Transform "ELzaki Transform" to Partial Differential Equations, *Global Journal of Pure and Applied Mathematics*, Vol.7, No.1, pp65-70.

Tarig M. Elzaki and Salih M. Elzaki,"On the Elzaki Transform and Ordinary Differential Equation with Variable Coefficients", *Advances in Theoretical and Applied Mathematics*. ISSN 0973-4554 Volume 6, Number 1(2011), pp. 13-18.

Tarig M. Elzaki, 2009. "Existence and Uniqueness of Solutions for Composite Type Equation", *Journal of Science and Technology*, pp. 214-219.

Tarig M. Elzaki, 2011. The New Integral Transform "ELzaki Transform", *Global Journal of Pure and Applied Mathematics*, Vol.7, No.1, pp57-64.

Tarig M. Elzaki, AdemKilicman, Hassan Eltayeb. 2010. " On Existence and Uniqueness of Generalized Solutions for a Mixed-Type Differential Equation*", Journal of Mathematics Research*, Vol. 2, No. 4, pp. 88-92.

Zafar, H. Khan, Waqar. A. Khan, *N-Transform- Properties and Applications,* NUST *Jour. Of Engg. Sciences*, 1(1):127-133, 2008.

Zain UlAbadin Zafar, *et al*., Solution of Burger's Equation with the help of Laplace *Decomposition method. Pak. J. Engg. & Appl. Sci.* 12: 39-42, 2013.

Zain UlAbadin Zafar, M.O. Ahmad, A. Pervaiz, Nazir Ahmad: ZZ Fourth Order Compact BVM for the Equation of Lateral Heat Loss ,*Pak. J. Engg. & Appl. Sci. Vol. 11, July.,*2012 (*p.* 96-103)

Zain UlAbadin Zafar, ZZ Transform method, IJAEGT, 4(1), 1605-1611, Jan (2016).

****