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TWO DIMENSIONAL FOURIER –MELLIN TRANSFORM OF SOME SIGNALS

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ABSTRACT

Thyroid Fourier Transform and Mellin Transform are mathematically related with each other. The Fourier-Mellin Transform combination is used for scale, rotation and translation properties. So, combination of this integral transforms have fabulous application in all field of Science. Fourier and Mellin's transformation has been applied in many different areas of physics and engineering. In the given paper, we have defined the Two Dimensional Fourier-Mellin Transform in the range [0,0,0,0] to $[\infty,\infty,a,b]$. The aim of this paper is to obtain the solution of some function using the Two Dimensional Fourier-Mellin Transform in the define range.

INTRODUCTION

The Mellin transform, a close relative of the Fourier Transform. The Mellin transform is related to the Fourier transform by a logarithmic coordinate transformation (Yunlong Sheng, 1986). It may come as a surprise to encounter the Fourier transform in a laboratory setting but in fact the Fourier transform is an essential tool of modern experimental physics and engineering. Many experiments in the Senior Physics Laboratory assume some familiarity with the Fourier transform. The Fourier integral transform is well known for finding the probability densities for sums and differences of random variables (Khairnar, 2012). Similarly, Mellin Transform is also very much important in all fields of science due its properties that we have less known. It has many applications such as quantum calculus, agriculture, medical stream, signal processing, optics, pattern recognition, radar classification of ships, electromagnetic, stress distribution, statistics, probability (Lokenath, 2007). This transform is vast of applications in computer science for analysis of algorithms, theory of functions, number theory, and partial differential equations (Dave Collins, ?; http://www-history.mcs.st-andrews.ac.uk/Biographies/Mellin.html and Flajolet, 1995). Mellin-transform (MT) method has application to antenna, electromagnetic problems. The Mellin Transform method is an extremely powerful technique for the exact evaluation of definite integrals (George Fikioris, 2006). Also, used in hyper geometric differential equations and to the derivation of asymptotic expansions (Bertrand, 2000). We use the Mellin integral transforms to derive different properties in statistics and probability densities of single continuous random variable (Khairnar, 2012). Fourier Transform and Mellin Transform are mathematically related with each other. The combination of these integral transform gives the important application in the field of applied sciences. The Fourier Mellin Transform (FMT) is used in the optical flow; the measurement of optical flow is one of the fundamental problems in computer vision. Fourier Mellin Transform is use in rigid image registration (Huy Tho Ho, 2006).

Definition of Two Dimensional Fourier-Mellin Transform (2DFMT)

The Conventional definition of 2DFMT using the previous paper (Sharma, 2011 and Sharma, 2014), definition is as-

$$FM\{f(t,l,x,y)\} = F(s,u,p,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,l,x,y)\, e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy \qquad \text{.........................(1.1)}$$

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Where, $K(t,l,x,y,s,u,p,v) = e^{-i(st+ul)}x^{p-1}y^{v-1}$ & $t(-\infty < t < \infty), l(-\infty < l < \infty), x(0 < x < \infty), y(0 < y < \infty)$.

In the present paper, we have defined Two Dimensional Fourier-Mellin Transform in the range [0,0,0,0] to $[\infty,\infty,a,b]$. Also, we have obtained the solution of some function using the Two Dimensional Fourier-Mellin Transform in the range [0,0,0,0] to$[\infty,\infty,a,b]$.

1. Definition of Two Dimensional Fourier-Mellin Transform (2DFMT) in the range $[\mathbf{0},\mathbf{0},\mathbf{0},\mathbf{0}]$ to $[\infty,\infty,\boldsymbol{a},\boldsymbol{b}]$

The Conventional definition of 2DFMT in the range [0,0,0,0] to $[\infty,\infty,a,b]$ using the previous paper [13, 14] definition is as

$$FM\{f(t,l,x,y)\} = F(s,u,p,v) = \int_0^\infty\int_0^\infty\int_0^a\int_0^b f(t,l,x,y)\, e^{-i(st+ul)}x^{p-1}y^{v-1}dt\,dl\,dx\,dy \quad \text{……………….(2.1)}$$

Where, $K(t,l,x,y,s,u,p,v) = e^{-i(st+ul)}x^{p-1}y^{v-1}$ & $t(0 < t < \infty), l(0 < l < \infty), x(0 < x < a), y(0 < y < b)$.

2. 2DFMT for some function in the range $[\mathbf{0},\mathbf{0},\mathbf{0},\mathbf{0}]$ to $[\boldsymbol{b},\boldsymbol{a},\infty,\infty]$

3.1. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then $FM\{1\} = -b^v a^p (supv)^{-1}$

Proof: We have from (2.1) as-

$$\begin{aligned} FM\{1\} &= \int_0^\infty\int_0^\infty\int_0^a\int_0^b 1\, e^{-i(st+ul)}x^{p-1}y^{v-1}dt\,dl\,dx\,dy \\ &= \int_0^\infty e^{-ist}\,dt \int_0^\infty e^{-iul}\,dl \quad \int_0^a x^{p-1}\,dx \int_0^b y^{v-1}\,dy \\ &= \left[\frac{e^{-ist}}{-is}\right]_0^\infty \left[\frac{e^{-iul}}{-iu}\right]_0^\infty \left[\frac{x^p}{p}\right]_0^a \left[\frac{y^v}{v}\right]_0^b \\ &= \left[\frac{-1}{-is}\right]\left[\frac{-1}{-iu}\right]\left[\frac{a^p}{p}\right]\left[\frac{b^v}{v}\right] \end{aligned}$$

$$FM\{1\} = \frac{-b^v a^p}{supv} = -b^v a^p (supv)^{-1}$$

3.2. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then $FM\{tlxy\} = b^{v+1}a^{p+1}[(p+1)(v+1)]^{-1}(su)^{-2}$

Proof: We have from (2.1) as

$$\begin{aligned} FM\{tlxy\} &= \int_0^\infty\int_0^\infty\int_0^a\int_0^b tlxy\, e^{-i(st+ul)}x^{p-1}y^{v-1}dt\,dl\,dx\,dy \\ &= \int_0^\infty te^{-ist}\,dt \int_0^\infty le^{-iul}\,dl \int_0^a x^p\,dx \int_0^b y^v\,dy \\ &= \left[\left(t\frac{e^{-ist}}{-is}\right)_0^\infty - \int_0^\infty \frac{e^{-ist}}{-is}dt\right]\left[\left(l\frac{e^{-iul}}{-iu}\right)_0^\infty - \int_0^\infty \frac{e^{-iul}}{-iu}dl\right]\left[\frac{x^{p+1}}{p+1}\right]_0^a \left[\frac{y^{v+1}}{v+1}\right]_0^b \\ &= \left[\frac{-1}{s^2}\right]\left[\frac{-1}{u^2}\right]\left[\frac{a^{p+1}}{p+1}\right]\left[\frac{b^{v+1}}{v+1}\right] \end{aligned}$$

$$FM\{tlxy\} = \frac{b^{v+1}a^{p+1}}{(p+1)(v+1)s^2u^2} = b^{v+1}a^{p+1}[(p+1)(v+1)]^{-1}(su)^{-2}$$

3.3. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then $FM\{t^n l^n x^n y^n\} = \frac{b^{v+n}a^{p+n}}{(-1)^{n+1}}[(p+n)(v+n)]^{-1}(su)^{-(n+1)}[n\Gamma n]^2$

Proof: We have from (2.1) as

$$FM\{t^n l^n x^n y^n\} = \int_0^\infty\int_0^\infty\int_0^a\int_0^b t^n l^n x^n y^n\, e^{-i(st+ul)}x^{p-1}y^{v-1}dt\,dl\,dx\,dy$$

$$\begin{aligned} FM\{t^n l^n x^n y^n\} &= \int_0^\infty t^n e^{-ist}\,dt \int_0^\infty l^n e^{-iul}\,dl \int_0^a x^{p+n-1}\,dx \int_0^b y^{v+n-1}\,dy \\ &= \left[\left(t^n\frac{e^{-ist}}{-is}\right)_0^\infty - n\int_0^\infty t^{n-1}\frac{e^{-ist}}{-is}dt\right]\left[\left(l^n\frac{e^{-iul}}{-iu}\right)_0^\infty - n\int_0^\infty l^{n-1}\frac{e^{-iul}}{-iu}dl\right]\left[\frac{x^{p+n}}{p+n}\right]_0^a \\ &\quad \left[\frac{y^{v+n}}{v+n}\right]_0^b \\ &= \left[\frac{n}{is}\int_0^\infty t^{n-1}e^{-ist}\,dt\right]\left[\frac{n}{iu}\int_0^\infty l^{n-1}e^{-ist}\,dt\right]\left[\frac{a^{p+n}}{p+n}\right]\left[\frac{b^{v+n}}{v+n}\right] \\ &= \left[\frac{1}{(is)^n}n(n-1)(n-2)\ldots\ldots 3.2.1\int_0^\infty e^{-ist}\,dt\right] \end{aligned}$$

$$\left[\frac{1}{(iu)^n} n(n-1)(n-2)\ldots\ldots 3.2.1 \int_0^\infty e^{-iul}\, dl\right]\left[\frac{a^{p+n}}{p+n}\right]\left[\frac{b^{v+n}}{v+n}\right]$$
$$= \left[\frac{1}{(is)^n}\, n! \left[\frac{e^{-ist}}{-is}\right]_0^\infty\right]\left[\frac{1}{(iu)^n} n! \left[\frac{e^{-iul}}{-iu}\right]_0^\infty\right]\left[\frac{a^{p+n}}{p+n}\right]\left[\frac{b^{v+n}}{v+n}\right]$$
$$= \frac{1}{(is)^{n+1}}\frac{1}{(iu)^{n+1}}\,[n!]^2\left[\frac{a^{p+n}}{p+n}\right]\left[\frac{b^{v+n}}{v+n}\right]$$
$$= \frac{b^{v+n}a^{p+n}}{(-1)^{n+1}(p+n)(v+n)(su)^{n+1}}\,[n\Gamma n]^2$$
$$FM\{t^n l^n x^n y^n\} = \frac{b^{v+n}a^{p+n}}{(-1)^{n+1}}\,[(p+n)(v+n)]^{-1}(su)^{-(n+1)}[n\Gamma n]^2$$

3.4. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then
$FM\{e^{at+bl}xy\} = b^{v+1}a^{p+1}[(p+1)(v+1)(is-a)(iu-b)]^{-1}$

Proof: - We have from (2.1) as-

$$FM\{e^{at+bl}xy\} = \int_0^\infty\int_0^\infty\int_0^a\int_0^b e^{at+bl}xy\, e^{-i(st+ul)}x^{p-1}y^{v-1}dt\, dl\, dx\, dy$$
$$= \int_0^\infty e^{-(is-a)t}\, dt \int_0^\infty e^{-(iu-b)l}\, dl \int_0^a x^p\, dx \int_0^b y^v\, dy$$
$$= \left[\frac{e^{-(is-a)t}}{-(is-a)}\right]_0^\infty \left[\frac{e^{-(iu-b)l}}{-(iu-b)}\right]_0^\infty \left[\frac{x^{p+1}}{p+1}\right]_0^a \left[\frac{y^{v+1}}{v+1}\right]_0^b$$
$$= \left[\frac{1}{(is-a)}\right]\left[\frac{1}{(iu-b)}\right]\left[\frac{a^{p+1}}{p+1}\right]\left[\frac{b^{v+1}}{v+1}\right]$$

$$FM\{e^{at+bl}xy\} = b^{v+1}a^{p+1}[(p+1)(v+1)(is-a)(iu-b)]^{-1}$$

3.5. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$FM\{\cos at \cos bl\, xy\} = -b^{v+1}a^{p+1}su[(v+1)(p+1)(s^2+a^2)(u^2+b^2)]^{-1}$

Proof: - We have from (2.1) as-

$$FM\{\cos at\cos bl\, xy\} = \int_0^\infty\int_0^\infty\int_0^a\int_0^b \cos at\cos bl\, xy\, e^{-i(st+ul)}x^{p-1}y^{v-1}dt\, dl\, dx\, dy$$
$$= \int_0^\infty \cos at\, e^{-ist}\, dt\int_0^\infty \cos bl\, e^{-iul}\, dl\int_0^a x^p\, dx\int_0^b y^v\, dy$$
$$= \left[\frac{e^{-ist}}{(s^2+a^2)}((-is)\cos at + a\sin at)\right]_0^\infty$$
$$\left[\frac{e^{-iul}}{(u^2+b^2)}((-iu)\cos bl + b\sin bl)\right]_0^\infty \left[\frac{x^{p+1}}{p+1}\right]_0^a \left[\frac{y^{v+1}}{v+1}\right]_0^b$$

$$= \left[\frac{is}{(s^2+a^2)}\right]\left[\frac{iu}{(u^2+b^2)}\right]\left[\frac{a^{p+1}}{p+1}\right]\left[\frac{b^{v+1}}{v+1}\right]$$

$$= \frac{-b^{v+1}a^{p+1}su}{(v+1)(p+1)(s^2+a^2)(u^2+b^2)}$$

$$FM\{\cos at\cos bl\, xy\} = -b^{v+1}a^{p+1}su[(v+1)(p+1)(s^2+a^2)(u^2+b^2)]^{-1}$$

3.6. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$FM\{\sin at \sin bl\, xy\} = b^{v+2}a^{p+2}[(v+1)(p+1)(s^2+a^2)(u^2+b^2)]^{-1}$

Proof: - We have from (2.1) as-

$$FM\{\sin at\sin bl\, xy\} = \int_0^\infty\int_0^\infty\int_0^a\int_0^b \sin at\sin bl\, xy\, e^{-i(st+ul)}x^{p-1}y^{v-1}dt\, dl\, dx\, dy$$

$$= \int_0^\infty \sin at\, e^{-ist}\, dt\int_0^\infty \sin bl\, e^{-iul}\, dl\int_0^a x^p\, dx\int_0^b y^v\, dy$$

$$= \left[\frac{e^{-ist}}{(s^2+a^2)}((-is)\sin at - a\cos at)\right]_0^\infty$$

$$\left[\frac{e^{-iul}}{(u^2+b^2)}((-iu)\sin bl - b\cos bl)\right]_0^\infty \left[\frac{x^{p+1}}{p+1}\right]_0^a \left[\frac{y^{v+1}}{v+1}\right]_0^b$$

$$= \left[\frac{a}{(s^2+a^2)}\right]\left[\frac{b}{(u^2+b^2)}\right]\left[\frac{a^{p+1}}{p+1}\right]\left[\frac{b^{v+1}}{v+1}\right]$$

$$= \frac{b^{v+2}a^{p+2}}{(v+1)(p+1)(s^2+a^2)(u^2+b^2)}$$

$$FM\{\sin at \sin bl\, xy\} = b^{v+2}a^{p+2}[(v+1)(p+1)(s^2+a^2)(u^2+b^2)]^{-1}$$

3.7. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then $FM\{\sin at \cos bl\, xy\ \} = (iu)b^{v+1}a^{p+2}[(v+1)(p+1)(s^2+a^2)(u^2+b^2)]^{-1}$

Proof: - We have from (2.1) as-

$$FM\{\sin at \cos bl\, xy\} = \int_0^\infty \int_0^\infty \int_0^a \int_0^b \sin at \cos bl\, xy\, e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$$

$$= \int_0^\infty \sin at\, e^{-ist}\ dt \int_0^\infty \cos bl\, e^{-iul}\ dl \int_0^a x^p\ dx \int_0^b y^v\, dy$$

$$= \left[\frac{e^{-ist}}{(s^2+a^2)}((-is)\sin at - a\cos at)\right]_0^\infty \left[\frac{e^{-iul}}{(u^2+b^2)}((-iu)\cos bl + b\sin bl)\right]_0^\infty$$

$$\left[\frac{x^{p+1}}{p+1}\right]_0^a \left[\frac{y^{v+1}}{v+1}\right]_0^b$$

$$= \left[\frac{a}{(s^2+a^2)}\right]\left[\frac{iu}{(u^2+b^2)}\right]\left[\frac{a^{p+1}}{p+1}\right]\left[\frac{b^{v+1}}{v+1}\right]$$

$$= \frac{b^{v+1}a^{p+2}(iu)}{(v+1)(p+1)(s^2+a^2)(u^2+b^2)}$$

$$FM\{\sin at \cos bl\, xy\} = (iu)b^{v+1}a^{p+2}[(v+1)(p+1)(s^2+a^2)(u^2+b^2)]^{-1}$$

3.8. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$$FM\{\cos at \sin bl\, xy\} = (is)b^{v+2}a^{p+1}[(v+1)(p+1)(s^2+a^2)(u^2+b^2)]^{-1}$$

Proof: - We have from (2.1) as-

$$FM\{\cos at \sin bl\, xy\} = \int_0^\infty \int_0^\infty \int_0^a \int_0^b \cos at \sin bl\, xy\, e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$$

$$= \int_0^\infty \cos at\, e^{-ist}\ dt \int_0^\infty \sin bl\, e^{-iul}\ dl \int_0^a x^p\ dx\ \int_0^b y^v\, dy$$

$$= \left[\frac{e^{-ist}}{(s^2+a^2)}((-is)\cos at + a\sin at)\right]_0^\infty$$

$$\left[\frac{e^{-iul}}{(u^2+b^2)}((-iu)\sin bl - b\cos bl)\right]_0^\infty \left[\frac{x^{p+1}}{p+1}\right]_0^a \left[\frac{y^{v+1}}{v+1}\right]_0^b$$

$$= \left[\frac{is}{(s^2+a^2)}\right]\left[\frac{b}{(u^2+b^2)}\right]\left[\frac{a^{p+1}}{p+1}\right]\left[\frac{b^{v+1}}{v+1}\right]$$

$$FM\{\cos at \sin bl\, xy\} = (is)b^{v+2}a^{p+1}[(v+1)(p+1)(s^2+a^2)(u^2+b^2)]^{-1}$$

3.9. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$$FM\{\cos at \cos bl\, x^n y^n\} = -b^{v+n}a^{p+n} su[(v+n)(p+n)(s^2+a^2)(u^2+b^2)]^{-1}$$

Proof: - We have from (2.1) as-

$$FM\{\cos at \cos bl\, x^n y^n\} = \int_0^\infty \int_0^\infty \int_0^a \int_0^b \cos at \cos bl\, x^n y^n\, e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$$

$$= \int_0^\infty \cos at\, e^{-ist}\ dt \int_0^\infty \cos bl\, e^{-iul}\ dl \int_0^a x^{p+n-1}\ dx\ \int_0^b y^{v+n-1}\, dy$$

$$= \left[\frac{e^{-ist}}{(s^2+a^2)}((-is)\cos at + a\sin at)\right]_0^\infty$$

$$\left[\frac{e^{-iul}}{(u^2+b^2)}((-iu)\cos bl + b\sin bl)\right]_0^\infty \left[\frac{x^{p+n}}{p+n}\right]_0^a \left[\frac{y^{v+n}}{v+n}\right]_0^b$$

$$= \left[\frac{is}{(s^2+a^2)}\right] \left[\frac{iu}{(u^2+b^2)}\right] \left[\frac{a^{p+n}}{p+n}\right] \left[\frac{b^{v+n}}{v+n}\right]$$

$FM\{\cos at \cos bl\, x^n y^n\ \} = -b^{v+n} a^{p+n} su[(v+n)(p+n)(s^2+a^2)(u^2+b^2)]^{-1}$

3.10. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$FM\{\sin at \sin bl\, x^n y^n\} = b^{v+n+1} a^{p+n+1} [(v+n)(p+n)(s^2+a^2)(u^2+b^2)]^{-1}$

Proof: - We have from (2.1) as-

$$FM\{\sin at \sin bl\, x^n y^n\} = \int_0^\infty \int_0^\infty \int_0^a \int_0^b \sin at \sin bl\, x^n y^n\, e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$$

$$= \int_0^\infty \sin at\, e^{-ist}\ dt \int_0^\infty \sin bl\, e^{-iul}\ dl \int_0^a x^{p+n-1}\ dx\ \int_0^b y^{v+n-1}\, dy$$

$$= \left[\frac{e^{-ist}}{(s^2+a^2)}((-is)\sin at - a\cos at)\right]_0^\infty$$

$$\left[\frac{e^{-iul}}{(u^2+b^2)}((-iu)\sin bl - b\cos bl)\right]_0^\infty \left[\frac{x^{p+n}}{p+n}\right]_0^a \left[\frac{y^{v+n}}{v+n}\right]_0^b$$

$$= \left[\frac{a}{(s^2+a^2)}\right] \left[\frac{b}{(u^2+b^2)}\right] \left[\frac{a^{p+n}}{p+n}\right] \left[\frac{b^{v+n}}{v+n}\right]$$

$$= \frac{b^{v+n+1} a^{p+n+1}}{(v+n)(p+n)(s^2+a^2)(u^2+b^2)}$$

$FM\{\cos at \cos bl\, xy\ \} = b^{v+n+1} a^{p+n+1} [(v+n)(p+n)(s^2+a^2)(u^2+b^2)]^{-1}$

3.11. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$FM\{\cosh at \cosh al\, xy\ \} = b^{v+1} a^{p+1} su[(v+1)(p+1)(s^2+a^2)(u^2+a^2)]^{-1}$

Proof: - We have from (2.1) as-

$$FM\{\cosh at \cosh bl\, xy\} = \int_0^\infty \int_0^\infty \int_0^a \int_0^b \cosh at \cosh bl\, xy\, e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$$

$$= \int_0^\infty \cosh at\, e^{-ist}\ dt \int_0^\infty \cosh al\, e^{-iul}\ dl \int_0^a x^p\ dx\ \int_0^b y^v\, dy$$

$$= \frac{1}{2}\left[\frac{e^{-(is-a)t}}{-(is-a)} + \frac{e^{-(is+a)t}}{-(is+a)}\right]_0^\infty \frac{1}{2}\left[\frac{e^{-(iu-a)l}}{-(iu-a)} + \frac{e^{-(iu+a)l}}{-(iu+a)}\right]_0^\infty \left[\frac{x^{p+1}}{p+1}\right]_0^a \left[\frac{y^{v+1}}{v+1}\right]_0^b$$

$$= \frac{1}{2}\left[\frac{2is}{(is)^2-a^2}\right] \frac{1}{2}\left[\frac{2iu}{(iu)^2-a^2}\right] \left[\frac{a^{p+1}}{p+1}\right] \left[\frac{b^{v+1}}{v+1}\right]$$

$$= \frac{-b^{v+1} a^{p+1} su}{(v+1)(p+1)(s^2+a^2)(u^2+a^2)}$$

$FM\{\cosh at \cosh al\, xy\ \} = b^{v+1} a^{p+1} su[(v+1)(p+1)(s^2+a^2)(u^2+a^2)]^{-1}$

3.12. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$FM\{\cosh at \sinh al\, xy\} = (is) b^{v+1} a^{p+2} [(v+1)(p+1)(s^2+a^2)(u^2+a^2)]^{-1}$

Proof: - We have from (2.1) as-

$$FM\{\cosh at \sinh al\, xy\} = \int_0^\infty \int_0^\infty \int_0^a \int_0^b \cosh at \sinh al\, xy\ \ e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$$

$$= \int_0^\infty \cosh at\, e^{-ist}\ dt \int_0^\infty \sinh al\, e^{-iul}\ dl \int_0^a x^p\ dx\ \int_0^b y^v\, dy$$

$$= \frac{1}{2}\left[\frac{e^{-(is-a)t}}{-(is-a)} + \frac{e^{-(is+a)t}}{-(is+a)}\right]_0^\infty \frac{1}{2}\left[\frac{e^{-(iu-a)l}}{-(iu-a)} - \frac{e^{-(iu+a)l}}{-(iu+a)}\right]_0^\infty \left[\frac{x^{p+1}}{p+1}\right]_0^a \left[\frac{y^{v+1}}{v+1}\right]_0^b$$

$$=\frac{1}{2}\left[\frac{2is}{(is)^2-a^2}\right]\frac{1}{2}\left[\frac{2a}{(iu)^2-a^2}\right]\left[\frac{a^{p+1}}{p+1}\right]\left[\frac{b^{v+1}}{v+1}\right]$$

$$=\frac{b^{v+1}a^{p+2}is}{(v+1)(p+1)(s^2+a^2)(u^2+a^2)}$$

$FM\{\cosh at \sinh al\, xy\} = (is)b^{v+1}a^{p+2}[(v+1)(p+1)(s^2+a^2)(u^2+a^2)]^{-1}$

3.13. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$FM\{\sinh at \sinh al\, xy\} = b^{v+1}a^{p+3}[(v+1)(p+1)(s^2+a^2)(u^2+a^2)]^{-1}$

Proof: - We have from (2.1) as-

$$FM\{\sinh at \sinh al\, xy\} = \int_0^\infty\int_0^\infty\int_0^a\int_0^b \sinh at \sinh al\, xy\, e^{-i(st+ul)}x^{p-1}y^{v-1}dt\, dl\, dx\, dy$$

$$=\int_0^\infty \sinh at\, e^{-ist}\ dt \int_0^\infty \sinh al\, e^{-iul}\ dl \int_0^a x^p\ dx \int_0^b y^v\, dy$$

$$=\frac{1}{2}\left[\frac{e^{-(is-a)t}}{-(is-a)}-\frac{e^{-(is+a)t}}{-(is+a)}\right]_0^\infty \frac{1}{2}\left[\frac{e^{-(iu-a)l}}{-(iu-a)}-\frac{e^{-(iu+a)l}}{-(iu+a)}\right]_0^\infty \left[\frac{x^{p+1}}{p+1}\right]_0^a\left[\frac{y^{v+1}}{v+1}\right]_0^b$$

$$=\frac{1}{2}\left[\frac{2a}{(is)^2-a^2}\right]\frac{1}{2}\left[\frac{2a}{(iu)^2-a^2}\right]\left[\frac{a^{p+1}}{p+1}\right]\left[\frac{b^{v+1}}{v+1}\right]$$

$$=\frac{b^{v+1}a^{p+3}}{(v+1)(p+1)(s^2+a^2)(u^2+a^2)}$$

$FM\{\cosh at \sinh bl\, xy\} = b^{v+1}a^{p+3}[(v+1)(p+1)(s^2+a^2)(u^2+a^2)]^{-1}$

4. The tabular form of above results obtained are given as

S.N.	$f(t,l,x,y)$	$FM\{f(t,l,x,y)\}$
1	1	$-b^v a^p (supv)^{-1}$
2	$tlxy$	$b^{v+1}a^{p+1}[(p+1)(v+1)]^{-1}(su)^{-2}$
3	$t^n l^n x^n y^n$	$\frac{b^{v+n}a^{p+n}}{(-1)^{n+1}}[(p+n)(v+n)]^{-1}(su)^{-(n+1)}[n\Gamma n]^2$
4	$e^{at+bl}xy$	$b^{v+1}a^{p+1}[(p+1)(v+1)(is-a)(iu-b)]^{-1}$
5	$cos\, at\, cos\, bl\, xy$	$-b^{v+1}a^{p+1}su[(v+1)(p+1)(s^2+a^2)(u^2+b^2)]^{-1}$
6	$sin\, at\, sin\, bl\, xy$	$b^{v+2}a^{p+2}[(v+1)(p+1)(s^2+a^2)(u^2+b^2)]^{-1}$
7	$\sin at \cos bl\, xy$	$(iu)b^{v+1}a^{p+2}[(v+1)(p+1)(s^2+a^2)(u^2+b^2)]^{-1}$
8	$\cos at \sin bl\, xy$	$(is)b^{v+2}a^{p+1}[(v+1)(p+1)(s^2+a^2)(u^2+b^2)]^{-1}$
9	$\cos at \cos bl\, x^n y^n$	$-b^{v+n}a^{p+n}su[(v+n)(p+n)(s^2+a^2)(u^2+b^2)]^{-1}$
10	$\sin at \sin bl\, x^n y^n$	$b^{v+n+1}a^{p+n+1}[(v+n)(p+n)(s^2+a^2)(u^2+b^2)]^{-1}$
11	$\cosh at \cosh al\, xy$	$b^{v+1}a^{p+1}su[(v+1)(p+1)(s^2+a^2)(u^2+a^2)]^{-1}$
12	$\cosh at \sinh al\, xy$	$(is)b^{v+1}a^{p+2}[(v+1)(p+1)(s^2+a^2)(u^2+a^2)]^{-1}$
13	$\sinh at \sinh al\, xy$	$b^{v+1}a^{p+3}[(v+1)(p+1)(s^2+a^2)(u^2+a^2)]^{-1}$

Conclusion

We have obtained the solution of different functions using the definition of Two Dimensional Fourier-Mellin Transform in range [0,0,0,0] to$[\infty,\infty,a,b]$.

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