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LAGRANGIAN MECHANICAL SYSTEMS ON THE STANDARD CLIFFORDIAN *KÄHLER* MANIFOLDS

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ABSTRACT

In this paper we obtained a canonical local basis $\{J_i\}$, $i = 1,2$ of vector bundle V on the Standard Cliffordian $K\ddot{a}hler$ Manifold (R^8, V). The paths of semispray on the Standard Cliffordian $K\ddot{a}hler$ Manifold are in fact the solutions of Euler-Lagrange equations.

INTRODUCTION

Modern differential geometry explains explicitly the dynamics of Lagrangians. So, we say that if M is an $m - dimensional$ configuration manifold and $L: TM \to R$ is a regular Lagrangian function, then there is a unique vector field ξ on TM such that dynamics equations is given by:

$$i_\xi \Phi_L = dE_L \tag{1}$$

Where Φ_L indicates the symplectic form and E_L is the energy associated to L.

The triple (TM, Φ_L, ξ) is called Lagrangian system on the tangent bundle TM.

In literature, there are a lot of studies about Lagrangian mechanics, formalisms, systems and equations [De Leon, 1989; Tekkoyun, 2005; Tekkoyun, 2005] and there in. There are real, complex, paracomplex and other analogues. It is possible to produce different analogues in different spaces. Finding new dynamics equations is both a new expansion and contribution to science to explain physical events.

Quaternions were invented by Sir William Rowan Hamilton as an extension to the complex number. Hamilton's defining relation is most succinctly written as:

$$i^2 = j^2 = k^2 = ijk = -1 \tag{2}$$

If it is compared to the calculus of vectors, quaternions have slipped into the realm of obscurity.

They do however still find use in the computation of rotations. A lot of physical laws in classical, relativistic, and quantum mechanics can be written pleasantly by means of quaternions. Some physicists hope they will find deeper understanding of the universe by restating basic principles in terms of quaternion algebra.

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It is well-known that quaternions are useful for representing rotations in both quantum and classical mechanics [http://www.stahlke.org/dan/Phys-papers/quaternion-paper.pdf]. Cliffordian manifold is a quaternion manifold. The above properties yield also for Cliffordian manifold.

Preliminaries

Throughout this paper, all mathematical objects and mappings are assumed to be smooth, i.e. infinitely differentiable and Einstein convention of summarizing is adopted. $\mathcal{F}(M), \mathcal{X}(M)$ and $\Lambda^1(M)$ denote the set of functions on M, the set of vector fields on M and the set of 1-forms on M, respectively.

Theorem

Let f be differentiable ϕ , ψ are 1-form,then [Abdulla Eid, 2008]:

- $d(f\phi) = df\wedge\phi + fd\phi$
- $d(\phi\wedge\psi) = d\phi\wedge\psi - \phi\wedge d\psi$

Definition (Kronecker's delta)

Kronecker's delta denote by δ and defined as follows [Joel, 2013; Liviu, 2009]:

$$\delta_i^j = \begin{cases} 1 \ ; & i = j \\ 0 \ ; & i \neq j \end{cases}$$

Cliffordian $K\ddot{a}hler$ Manifolds

Here, we recalled the main concepts and structures given in [Yano, 1984; Burdujan, 2008]. Let M be a real smooth manifold of dimension m. Suppose that there is a 6-dimensional vector bundle V consisting of $F_i (i = 1,2, \dots ,6)$ tensors of type (1,1) over M. Such a local basis $\{F_1, F_2, \dots , F_6\}$ is called a canonical local basis of the bundle V in neighborhood $U\ of\ M$. Then V is called an almost Cliffordian structure in M. The pair (M, V) is named an almost Cliffordian manifold with V. Hence, an almost Cliffordian manifold M is of $dimension\ m = 8n$. If there exists on (M, V) a global basis $\{F_1, F_2, \dots , F_6\}$ $, then$ (M, V) is said to be an almost Cliffordian manifold; the basis $\{F_1, F_2, \dots , F_6\}$ is called a global basis for V.

An almost Cliffordian connection on the almost Cliffordian manifold(M, V) is a linear connection $\nabla\ on\ M$ which preserves by parallel transport the vector bundle V. This means that if Φ is a cross-section (local-global) of the bundle V $, then$ $\nabla_X\Phi$ is also a cross-section (local-global, respectively) of V , X being an arbitrary vector field of M.

If for any canonical basis $\{J_1, J_2, \dots , J_6\}\ of\ V$ in a coordinate neighborhood , the identities

$$g(J_iX , J_iY) = g(X , Y),\ \forall X, Y \in \mathcal{X}(M) ,\ i = 1,2, \dots ,6 \tag{3}$$

Hold, the triple (M, g, V) is named an almost Cliffordian Hermitian manifold or metric Cliffordian denoting by V an almost Cliffordian structure $V\ and\ by\ g$ a Riemannian metric and by (g, V) an almost Cliffordian metric structure.

Since each $J_i (i = 1,2, \dots ,6)$ is almost Hermitian structure with respect to g, setting

$$\Phi_i(X, Y) = g(J_iX, Y), \quad i = 1,2, \dots ,6 \tag{4}$$

For any vector fields $\ and\ Y$, we see that Φ_i are 6 local 2-forms.

If the Levi-Civita connection $\nabla = \nabla^g\ on\ (M, g, V)$ preserves the vector bundle V by parallel transport, then (M, g, V) is called a Cliffordian $K\ddot{a}hler$ manifold, and an almost Cliffordian structure $\Phi_i\ of\ M$ is called a Cliffordian $K\ddot{a}hler$ structure. A Clifford $K\ddot{a}hler$ manifold is Riemannian manifold (M^{8n}, g). For example, we say that R^{8n} is the simplest example of Clifford $K\ddot{a}hler$ manifold. Suppose that let $\{x_i, x_{n+i} x_{2n+i}, x_{3n+i}, x_{4n+i}, x_{5n+i}, x_{6n+i}, x_{7n+i}\}$, $i = \overline{1 , n}$ be a real coordinate system on R^{8n}. Then we define by $\left\{\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_{n+i}}, \frac{\partial}{\partial x_{2n+i}}, \frac{\partial}{\partial x_{3n+i}}, \frac{\partial}{\partial x_{4n+i}}, \frac{\partial}{\partial x_{5n+i}}, \frac{\partial}{\partial x_{6n+i}}, \frac{\partial}{\partial x_{7n+i}}\right\}$ and $\{dx_i, dx_{n+i}, dx_{2n+i}, dx_{3n+i}, dx_{4n+i}, dx_{5n+i}, dx_{6n+i}, dx_{7n+i}\}$ be natural bases over R of the tangent space $T(R^{8n})$ and the cotangent space $T^*(R^{8n})\ of\ R^{8n}$, respectively.

By structures J_1 , J_2 , J_3, the following expressions are obtained

$$J_1\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial x_{n+i}} \qquad J_2\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial x_{2n+i}} \qquad J_3\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial x_{3n+i}}$$

$$J_1\left(\frac{\partial}{\partial x_{n+i}}\right) = -\frac{\partial}{\partial x_i} \qquad J_2\left(\frac{\partial}{\partial x_{n+i}}\right) = -\frac{\partial}{\partial x_{4n+i}} \qquad J_3\left(\frac{\partial}{\partial x_{n+i}}\right) = -\frac{\partial}{\partial x_{5n+i}}$$

$$J_1\left(\frac{\partial}{\partial x_{2n+i}}\right) = \frac{\partial}{\partial x_{4n+i}} \qquad J_2\left(\frac{\partial}{\partial x_{2n+i}}\right) = -\frac{\partial}{\partial x_i} \qquad J_3\left(\frac{\partial}{\partial x_{2n+i}}\right) = -\frac{\partial}{\partial x_{6n+i}}$$

$$J_1\left(\frac{\partial}{\partial x_{i+3n}}\right)=\frac{\partial}{\partial x_{i+5n}} \quad J_2\left(\frac{\partial}{\partial x_{i+3n}}\right)=\frac{\partial}{\partial x_{i+6n}} \quad J_3\left(\frac{\partial}{\partial x_{i+3n}}\right)=-\frac{\partial}{\partial x_{i}}$$
$$J_1\left(\frac{\partial}{\partial x_{i+4n}}\right)=-\frac{\partial}{\partial x_{i+2n}} \quad J_2\left(\frac{\partial}{\partial x_{i+4n}}\right)=\frac{\partial}{\partial x_{i+n}} \quad J_3\left(\frac{\partial}{\partial x_{i+4n}}\right)=\frac{\partial}{\partial x_{i+7n}}$$
$$J_1\left(\frac{\partial}{\partial x_{i+5n}}\right)=-\frac{\partial}{\partial x_{i+3n}} \quad J_2\left(\frac{\partial}{\partial x_{i+5n}}\right)=-\frac{\partial}{\partial x_{i+7n}} \quad J_3\left(\frac{\partial}{\partial x_{i+5n}}\right)=\frac{\partial}{\partial x_{i+n}}$$
$$J_1\left(\frac{\partial}{\partial x_{6n+i}}\right)=\frac{\partial}{\partial x_{7n+i}} \quad J_2\left(\frac{\partial}{\partial x_{6n+i}}\right)=-\frac{\partial}{\partial x_{3n+i}} \quad J_3\left(\frac{\partial}{\partial x_{6n+i}}\right)=\frac{\partial}{\partial x_{2n+i}}$$
$$J_1\left(\frac{\partial}{\partial x_{7n+i}}\right)=-\frac{\partial}{\partial x_{6n+i}} \quad J_2\left(\frac{\partial}{\partial x_{7n+i}}\right)=\frac{\partial}{\partial x_{5n+i}} \quad J_3\left(\frac{\partial}{\partial x_{7n+i}}\right)=-\frac{\partial}{\partial x_{4n+i}} \qquad (5)$$

Lagrangian Mechanics

In this section, we obtain Euler-Lagrange equations for quantum and classical mechanics by means of a canonical local basis $\{J_1, J_2, J_3\}$ $of\ V$ on the standard Cliffordian $Kähler$ manifold (R^{8n}, V).

First:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right)+\frac{\partial L}{\partial x_{n+i}}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right)-\frac{\partial L}{\partial x_i}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right)+\frac{\partial L}{\partial x_{4n+i}}=0,$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right)+\frac{\partial L}{\partial x_{5n+i}}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right)-\frac{\partial L}{\partial x_{2n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right)-\frac{\partial L}{\partial x_{3n+i}}=0,$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right)+\frac{\partial L}{\partial x_{7n+i}}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right)-\frac{\partial L}{\partial x_{6n+i}}=0.$$

Second, let J_2 take a local basis component on the standard Cliffordian $Kähler$ manifold (R^{8n}, V) and $\{x_i, x_{n+i}x_{2n+i}, x_{3n+i}, x_{4n+i}, x_{5n+i}, x_{6n+i}, x_{7n+i}\}$, $i=\overline{1,n}$ be its coordinate functions.

Let semispray be the vector field ξ determined by:

$$\xi = X^i\frac{\partial}{\partial x_i}+X^{n+i}\frac{\partial}{\partial x_{n+i}}+X^{2n+i}\frac{\partial}{\partial x_{2n+i}}+X^{3n+i}\frac{\partial}{\partial x_{3n+i}}+X^{4n+i}\frac{\partial}{\partial x_{4n+i}}$$
$$+X^{5n+i}\frac{\partial}{\partial x_{5n+i}}+X^{6n+i}\frac{\partial}{\partial x_{6n+i}}+X^{7n+i}\frac{\partial}{\partial x_{7n+i}} \qquad (6)$$

Where

$$X^i=\dot{x}_i\,,X^{n+i}=\dot{x}_{n+i}\,,X^{2n+i}=\dot{x}_{2n+i}\,,X^{3n+i}=\dot{x}_{3n+i}\,,X^{4n+i}=\dot{x}_{4n+i}$$
$$X^{5n+i}=\dot{x}_{5n+i}\,,X^{6n+i}=\dot{x}_{6n+i}\,,X^{7n+i}=\dot{x}_{7n+i}.$$

This equation (6) can be written concise manner

$$\xi=\sum_{a=0}^{7}X^{an+i}\frac{\partial}{\partial x_{an+i}} \qquad (7)$$

And the dot indicates the derivative with respect to time t. The vector field defined by

$$V_{J_2}=J_2(\xi)=X^i\frac{\partial}{\partial x_{2n+i}}-X^{n+i}\frac{\partial}{\partial x_{4n+i}}-X^{2n+i}\frac{\partial}{\partial x_i}+X^{3n+i}\frac{\partial}{\partial x_{6n+i}}+X^{4n+i}\frac{\partial}{\partial x_{n+i}}$$
$$-\ X^{5n+i}\frac{\partial}{\partial x_{7n+i}}-\ X^{6n+i}\frac{\partial}{\partial x_{3n+i}}+\ X^{7n+i}\frac{\partial}{\partial x_{5n+i}} \qquad (8)$$

Is called Liouville vector field on the standard Cliffordian $Kähler$ manifold (R^{8n}, V). The maps given by $T\,,P\colon R^{8n}\rightarrow R$ such that:

$$T=\frac{1}{2}m_i(\dot{x}_i^2+\dot{x}_{n+i}^2+\dot{x}_{2n+i}^2+\dot{x}_{3n+i}^2+\dot{x}_{4n+i}^2+\dot{x}_{5n+i}^2+\dot{x}_{6n+i}^2+\dot{x}_{7n+i}^2)$$

$$\therefore\ T=\frac{1}{2}m_i\sum_{a=0}^{7}\dot{x}_{an+i}^2 \qquad ,\quad P=m_igh$$

Are called the kinetic energy and the potential energy of the system, respectively. Here m_i , $g\ and\ h$ stand for mass of a mechanical system having m particles, the gravity acceleration and distance to the origin of a mechanical system on the standard Cliffordian $Kähler$ manifold (R^{8n}, V), respectively.

Then $L\colon R^{8n}\rightarrow R$ is a map that satisfies the conditions:

i) $L=T-P$ is a Lagrangian function.
ii) the function given by $E_L^{J_2}=V_{J_2}(L)-L$, is energy function.

The operator i_{J_2} induced by J_2 and given by:

$$i_{J_2}\omega(X_1, X_2, \dots, X_r) = \sum_{i=1}^{r} \omega\,(X_1, \dots, J_2X_i, \dots, X_r) \tag{9}$$

Is said to be vertical derivation, where $\omega \in \Lambda^r R^{8n}$, $X_i \in \mathcal{X}(R^{8n})$. The vertical differentiation d_{J_2} is defined by:

$$d_{J_2} = [i_{J_2}, d] = i_{J_2}d - di_{J_2} \tag{10}$$

Where d is the usual exterior derivation. For J_2, the closed Cliffordian $Kähler$ form is the closed 2-form given by $\Phi_L^{J_2} = -dd_{J_2}L$ such that

$$d_{J_2} = \frac{\partial}{\partial x_{2n+i}}dx_i - \frac{\partial}{\partial x_{4n+i}}dx_{n+i} - \frac{\partial}{\partial x_i}dx_{2n+i} + \frac{\partial}{\partial x_{6n+i}}dx_{3n+i} + \frac{\partial}{\partial x_{n+i}}dx_{4n+i} - \frac{\partial}{\partial x_{7n+i}}dx_{5n+i} - \frac{\partial}{\partial x_{3n+i}}dx_{6n+i} + \frac{\partial}{\partial x_{5n+i}}dx_{7n+i}$$

Defined by operator

$$d_{J_2} : \mathcal{F}(M) \rightarrow \Lambda^1 R^{8n} \tag{11}$$

Then

$$\begin{aligned}
\Phi_L^{J_2} = &-\frac{\partial^2 L}{\partial x_j \partial x_{2n+i}}dx_j \wedge dx_i + \frac{\partial^2 L}{\partial x_j \partial x_{4n+i}}dx_j \wedge dx_{n+i} + \frac{\partial^2 L}{\partial x_j \partial x_i}dx_j \wedge dx_{2n+i} \\
&- \frac{\partial^2 L}{\partial x_j \partial x_{6n+i}}dx_j \wedge dx_{3n+i} - \frac{\partial^2 L}{\partial x_j \partial x_{n+i}}dx_j \wedge dx_{4n+i} + \frac{\partial^2 L}{\partial x_j \partial x_{7n+i}}dx_j \wedge dx_{5n+i} \\
&+ \frac{\partial^2 L}{\partial x_j \partial x_{3n+i}}dx_j \wedge dx_{6n+i} - \frac{\partial^2 L}{\partial x_j \partial x_{5n+i}}dx_j \wedge dx_{7n+i} - \frac{\partial^2 L}{\partial x_{n+j} \partial x_{2n+i}}dx_{n+j} \wedge dx_i \\
&+ \frac{\partial^2 L}{\partial x_{n+j} \partial x_{4n+i}}dx_{n+j} \wedge dx_{n+i} + \frac{\partial^2 L}{\partial x_{n+j} \partial x_i}dx_{n+j} \wedge dx_{2n+i} - \frac{\partial^2 L}{\partial x_{n+j} \partial x_{6n+i}}dx_{n+j} \wedge dx_{3n+i} \\
&- \frac{\partial^2 L}{\partial x_{n+j} \partial x_{n+i}}dx_{n+j} \wedge dx_{4n+i} + \frac{\partial^2 L}{\partial x_{n+j} \partial x_{7n+i}}dx_{n+j} \wedge dx_{5n+i} + \frac{\partial^2 L}{\partial x_{n+j} \partial x_{3n+i}}dx_{n+j} \wedge dx_{6n+i} \\
&- \frac{\partial^2 L}{\partial x_{n+j} \partial x_{5n+i}}dx_{n+j} \wedge dx_{7n+i} - \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{2n+i}}dx_{2n+j} \wedge dx_i + \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{4n+i}}dx_{2n+j} \wedge dx_{n+i} + \\
&\frac{\partial^2 L}{\partial x_{2n+j} \partial x_i}dx_{2n+j} \wedge dx_{2n+i} - \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{6n+i}}dx_{2n+j} \wedge dx_{3n+i} - \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{n+i}}dx_{2n+j} \wedge dx_{4n+i} + \\
&\frac{\partial^2 L}{\partial x_{2n+j} \partial x_{7n+i}}dx_{2n+j} \wedge dx_{5n+i} + \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{3n+i}}dx_{2n+j} \wedge dx_{6n+i} - \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{5n+i}}dx_{2n+j} \wedge dx_{7n+i} \\
&- \frac{\partial^2 L}{\partial x_{3n+j} \partial x_i}dx_{3n+j} \wedge dx_i + \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{4n+i}}dx_{3n+j} \wedge dx_{n+i} + \frac{\partial^2 L}{\partial x_{3n+j} \partial x_i}dx_{3n+j} \wedge dx_{2n+i} \\
&- \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{6n+i}}dx_{3n+j} \wedge dx_{3n+i} - \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{n+i}}dx_{3n+j} \wedge dx_{4n+i} + \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{7n+i}}dx_{3n+j} \wedge dx_{5n+i} \\
&+ \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{3n+i}}dx_{3n+j} \wedge dx_{6n+i} - \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{5n+i}}dx_{3n+j} \wedge dx_{7n+i} - \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{2n+i}}dx_{4n+j} \wedge dx_i \\
&+ \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{4n+i}}dx_{4n+j} \wedge dx_{n+i} + \frac{\partial^2 L}{\partial x_{4n+j} \partial x_i}dx_{4n+j} \wedge dx_{2n+i} - \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{6n+i}}dx_{4n+j} \wedge dx_{3n+i} \\
&- \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{n+i}}dx_{4n+j} \wedge dx_{4n+i} + \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{7n+i}}dx_{4n+j} \wedge dx_{5n+i} + \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{3n+i}}dx_{4n+j} \wedge dx_{6n+i} \\
&- \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{5n+i}}dx_{4n+j} \wedge dx_{7n+i} - \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{2n+i}}dx_{5n+j} \wedge dx_i + \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{4n+i}}dx_{5n+j} \wedge dx_{n+i} \\
&+ \frac{\partial^2 L}{\partial x_{5n+j} \partial x_i}dx_{5n+j} \wedge dx_{2n+i} - \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{6n+i}}dx_{5n+j} \wedge dx_{3n+i} - \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{n+i}}dx_{5n+j} \wedge dx_{4n+i} \\
&+ \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{7n+i}}dx_{5n+j} \wedge dx_{5n+i} + \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{3n+i}}dx_{5n+j} \wedge dx_{6n+i} - \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{5n+i}}dx_{5n+j} \wedge dx_{7n+i} - \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{2n+i}}dx_{6n+j} \wedge dx_i + \\
&\frac{\partial^2 L}{\partial x_{6n+j} \partial x_{4n+i}}dx_{6n+j} \wedge dx_{n+i} + \frac{\partial^2 L}{\partial x_{6n+j} \partial x_i}dx_{6n+j} \wedge dx_{2n+i} \\
&- \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{6n+i}}dx_{6n+j} \wedge dx_{3n+i} - \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{n+i}}dx_{6n+j} \wedge dx_{4n+i} + \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{7n+i}}dx_{6n+j} \wedge dx_{5n+i} \\
&+ \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{3n+i}}dx_{6n+j} \wedge dx_{6n+i} - \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{5n+i}}dx_{6n+j} \wedge dx_{7n+i} - \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{2n+i}}dx_{7n+j} \wedge dx_i + \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{4n+i}}dx_{7n+j} \wedge dx_{n+i} + \\
&\frac{\partial^2 L}{\partial x_{7n+j} \partial x_i}dx_{7n+j} \wedge dx_{2n+i} - \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{6n+i}}dx_{7n+j} \wedge dx_{3n+i} \\
&- \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{n+i}}dx_{7n+j} \wedge dx_{4n+i} + \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{7n+i}}dx_{7n+j} \wedge dx_{5n+i} + \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{3n+i}}dx_{7n+j} \wedge dx_{6n+i} \\
&- \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{5n+i}}dx_{7n+j} \wedge dx_{7n+i}
\end{aligned}$$

Let ξ be the second order differential equation by given Eq(1) and defined by Eq(6) and

$$i_\xi \Phi_L^{J_2} = -X^i \frac{\partial^2 L}{\partial x_j \partial x_{2n+i}} \delta_i^j dx_i + X^i \frac{\partial^2 L}{\partial x_j \partial x_{2n+i}} dx_j + X^i \frac{\partial^2 L}{\partial x_j \partial x_{4n+i}} \delta_i^j dx_{n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_j \partial x_{4n+i}} dx_j$$

$$+X^i \frac{\partial^2 L}{\partial x_j \partial x_i} \delta_i^j dx_{2n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_j \partial x_i} dx_j - X^i \frac{\partial^2 L}{\partial x_j \partial x_{6n+i}} \delta_i^j dx_{3n+i} + X^{3n+i} \frac{\partial^2 L}{\partial x_j \partial x_{6n+i}} dx_j$$

$$-X^i \frac{\partial^2 L}{\partial x_j \partial x_{n+i}} \delta_i^j dx_{4n+i} + X^{4n+i} \frac{\partial^2 L}{\partial x_j \partial x_{n+i}} dx_j + X^i \frac{\partial^2 L}{\partial x_j \partial x_{7n+i}} \delta_i^j dx_{5n+i} - X^{5n+i} \frac{\partial^2 L}{\partial x_j \partial x_{7n+i}} dx_j +$$

$$X^i \frac{\partial^2 L}{\partial x_j \partial x_{3n+i}} \delta_i^j dx_{6n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_j \partial x_{3n+i}} dx_j - X^i \frac{\partial^2 L}{\partial x_j \partial x_{5n+i}} \delta_i^j dx_{7n+i} + X^{7n+i} \frac{\partial^2 L}{\partial x_j \partial x_{5n+i}} dx_j$$

$$-X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{2n+i}} \delta_{n+i}^{n+j} dx_i + X^i \frac{\partial^2 L}{\partial x_{n+j} \partial x_{2n+i}} dx_{n+j} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{4n+i}} \delta_{n+i}^{n+j} dx_{n+i}$$

$$- X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{4n+i}} dx_{n+j} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_i} \delta_{n+i}^{n+j} dx_{2n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_i} dx_{n+j} -$$

$$X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{6n+i}} \delta_{n+i}^{n+j} dx_{3n+i} + X^{3n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{6n+i}} dx_{n+j} - X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{n+i}} \delta_{n+i}^{n+j} dx_{4n+i}$$

$$+ X^{4n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{n+i}} dx_{n+j} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{7n+i}} \delta_{n+i}^{n+j} dx_{5n+i} - X^{5n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{7n+i}} dx_{n+j}$$

$$+X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{3n+i}} \delta_{n+i}^{n+j} dx_{6n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{3n+i}} dx_{n+j} - X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{5n+i}} \delta_{n+i}^{n+j} dx_{7n+i}$$

$$+X^{7n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{5n+i}} dx_{n+j} - X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{2n+i}} \delta_{2n+i}^{2n+j} dx_i + X^i \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{2n+i}} dx_{2n+j} +$$

$$X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{4n+i}} \delta_{2n+i}^{2n+j} dx_{n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{4n+i}} dx_{2n+j} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_i} \delta_{2n+i}^{2n+j} dx_{2n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_i} dx_{2n+j} -$$

$$X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{6n+i}} \delta_{2n+i}^{2n+j} dx_{3n+i} + X^{3n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{6n+i}} dx_{2n+j}$$

$$-X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{n+i}} \delta_{2n+i}^{2n+j} dx_{4n+i} + X^{4n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{n+i}} dx_{2n+j} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{7n+i}} \delta_{2n+i}^{2n+j} dx_{5n+i}$$

$$-X^{5n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{7n+i}} dx_{2n+j} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{3n+i}} \delta_{2n+i}^{2n+j} dx_{6n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{3n+i}} dx_{2n+j}$$

$$- X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{5n+i}} \delta_{2n+i}^{2n+j} dx_{7n+i} + X^{7n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{5n+i}} dx_{2n+j} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{2n+i}} \delta_{3n+i}^{3n+j} dx_i$$

$$+X^i \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{2n+i}} dx_{3n+j} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{4n+i}} \delta_{3n+i}^{3n+j} dx_{n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{4n+i}} dx_{3n+j} +$$

$$X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_i} \delta_{3n+i}^{3n+j} dx_{2n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_i} dx_{3n+j} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{6n+i}} \delta_{3n+i}^{3n+j} dx_{3n+i}$$

$$+ X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{6n+i}} dx_{3n+j} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{n+i}} \delta_{3n+i}^{3n+j} dx_{4n+i} + X^{4n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{n+i}} dx_{3n+j} +$$

$$X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{7n+i}} \delta_{3n+i}^{3n+j} dx_{5n+i} - X^{5n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{7n+i}} dx_{3n+j} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{3n+i}} \delta_{3n+i}^{3n+j} dx_{6n+i}$$

$$-X^{6n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{3n+i}} dx_{3n+j} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{5n+i}} \delta_{3n+i}^{3n+j} dx_{7n+i} + X^{7n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{5n+i}} dx_{3n+j} - X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{2n+i}} \delta_{4n+i}^{4n+j} dx_i +$$

$$X^i \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{2n+i}} dx_{4n+j} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{4n+i}} \delta_{4n+i}^{4n+j} dx_{n+i}$$

$$- X^{n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{4n+i}} dx_{4n+j} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_i} \delta_{4n+i}^{4n+j} dx_{2n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_i} dx_{4n+j}$$

$$-X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{6n+i}} \delta_{4n+i}^{4n+j} dx_{3n+i} + X^{3n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{6n+i}} dx_{4n+j} - X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{n+i}} \delta_{4n+i}^{4n+j} dx_{4n+i}$$

$$+X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{n+i}} dx_{4n+j} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{7n+i}} \delta_{4n+i}^{4n+j} dx_{5n+i} - X^{5n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{7n+i}} dx_{4n+j} +$$

$$X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{3n+i}} \delta_{4n+i}^{4n+j} dx_{6n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{3n+i}} dx_{4n+j} - X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{5n+i}} \delta_{4n+i}^{4n+j} dx_{7n+i}$$

$$+X^{7n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{5n+i}} dx_{4n+j} - X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{2n+i}} \delta_{5n+i}^{5n+j} dx_i + X^i \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{2n+i}} dx_{5n+j} +$$

$$X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{4n+i}} \delta_{5n+i}^{5n+j} dx_{n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{4n+i}} dx_{5n+j} + X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_i} \delta_{5n+i}^{5n+j} dx_{2n+i}$$

$$-X^{2n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_i} dx_{5n+j} - X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{6n+i}} \delta_{5n+i}^{5n+j} dx_{3n+i} + X^{3n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{6n+i}} dx_{5n+j}$$

$$-X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{n+i}} \delta_{5n+i}^{5n+j} dx_{4n+i} + X^{4n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{n+i}} dx_{5n+j} + X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{7n+i}} \delta_{5n+i}^{5n+j} dx_{5n+i}$$

$$-X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{7n+i}} dx_{5n+j} + X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{3n+i}} \delta_{5n+i}^{5n+j} dx_{6n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{3n+i}} dx_{5n+j}$$

$$-X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{5n+i}} \delta_{5n+i}^{5n+j} dx_{7n+i} + X^{7n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{5n+i}} dx_{5n+j} - X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{2n+i}} \delta_{6n+i}^{6n+j} dx_i$$

$$+X^i \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{2n+i}} dx_{6n+j} + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{4n+i}} \delta_{6n+i}^{6n+j} dx_{n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{4n+i}} dx_{6n+j}$$

$$+X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_i} \delta_{6n+i}^{6n+j} dx_{2n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_i} dx_{6n+j} - X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{6n+i}} \delta_{6n+i}^{6n+j} dx_{3n+i}$$

$$+X^{3n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{6n+i}} dx_{6n+j} - X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{n+i}} \delta_{6n+i}^{6n+j} dx_{4n+i} + X^{4n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{n+i}} dx_{6n+j} +$$

$$X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{7n+i}} \delta_{6n+i}^{6n+j} dx_{5n+i} - X^{5n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{7n+i}} dx_{6n+j} + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{3n+i}} \delta_{6n+i}^{6n+j} dx_{6n+i}$$

$$-X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}dx_{6n+j} - X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}\delta_{6n+i}^{6n+j}dx_{7n+i} + X^{7n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}dx_{6n+j}$$
$$-X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}\delta_{7n+i}^{7n+j}dx_i + X^i\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}dx_{7n+j} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}\delta_{7n+i}^{7n+j}dx_{n+i}$$
$$-X^{n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}dx_{7n+j} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_i}\delta_{7n+i}^{7n+j}dx_{2n+i} - X^{2n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_i}dx_{7n+j}$$
$$-X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}\delta_{7n+i}^{7n+j}dx_{3n+i} + X^{3n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}dx_{7n+j} - X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}\delta_{7n+i}^{7n+j}dx_{4n+i}$$
$$+X^{4n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}dx_{7n+j} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}\delta_{7n+i}^{7n+j}dx_{5n+i} - X^{5n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}dx_{7n+j} +$$
$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}\delta_{7n+i}^{7n+j}dx_{6n+i} - X^{6n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}dx_{7n+j} - X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}\delta_{7n+i}^{7n+j}dx_{7n+i}$$
$$+ X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}dx_{7n+j}$$

Since the closed standard Cliffordian $K\ddot{a}hler$ form $\Phi_L^{J_2}$ on (R^{8n}, V) is the symplectic structure, it is found

$$E_L^{J_2} = V_{J_2}(L) - L = X^i\frac{\partial L}{\partial x_{2n+i}} - X^{n+i}\frac{\partial L}{\partial x_{4n+i}} - X^{2n+i}\frac{\partial L}{\partial x_i} + X^{3n+i}\frac{\partial L}{\partial x_{6n+i}} + X^{4n+i}\frac{\partial L}{\partial x_{n+i}} - X^{5n+i}\frac{\partial L}{\partial x_{7n+i}} - X^{6n+i}\frac{\partial L}{\partial x_{3n+i}} + X^{7n+i}\frac{\partial L}{\partial x_{5n+i}} - L \quad (12)$$

And hence

$$dE_L^{J_2} = X^i\frac{\partial^2 L}{\partial x_j\partial x_{2n+i}}dx_j - X^{n+i}\frac{\partial^2 L}{\partial x_j\partial x_{4n+i}}dx_j - X^{2n+i}\frac{\partial^2 L}{\partial x_j\partial x_i}dx_j + X^{3n+i}\frac{\partial^2 L}{\partial x_j\partial x_{6n+i}}dx_j$$
$$+X^{4n+i}\frac{\partial^2 L}{\partial x_j\partial x_{n+i}}dx_j - X^{5n+i}\frac{\partial^2 L}{\partial x_j\partial x_{7n+i}}dx_j - X^{6n+i}\frac{\partial^2 L}{\partial x_j\partial x_{3n+i}}dx_j + X^{7n+i}\frac{\partial^2 L}{\partial x_j\partial x_{5n+i}}dx_j$$
$$+X^i\frac{\partial^2 L}{\partial x_{n+j}\partial x_{2n+i}}dx_{n+j} - X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{4n+i}}dx_{n+j} - X^{2n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_i}dx_{n+j} +$$
$$X^{3n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{6n+i}}dx_{n+j} + X^{4n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{n+i}}dx_{n+j} - X^{5n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{7n+i}}dx_{n+j} -$$
$$X^{6n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{3n+i}}dx_{n+j} + X^{7n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{5n+i}}dx_{n+j} + X^i\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{2n+i}}dx_{2n+j}$$
$$-X^{n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{4n+i}}dx_{2n+j} - X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_i}dx_{2n+j} + X^{3n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{6n+i}}dx_{2n+j}$$
$$+X^{4n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{n+i}}dx_{2n+j} - X^{5n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{7n+i}}dx_{2n+j} - X^{6n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{3n+i}}dx_{2n+j}$$
$$+X^{7n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{5n+i}}dx_{2n+j} + X^i\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{2n+i}}dx_{3n+j} - X^{n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{4n+i}}dx_{3n+j}$$
$$-X^{2n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_i}dx_{3n+j} + X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}}dx_{3n+j} + X^{4n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}}dx_{3n+j}$$
$$-X^{5n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}}dx_{3n+j} - X^{6n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}}dx_{3n+j} + X^{7n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}}dx_{3n+j}$$
$$+X^i\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{2n+i}}dx_{4n+j} - X^{n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{4n+i}}dx_{4n+j} - X^{2n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_i}dx_{4n+j}$$
$$+ X^{3n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}}dx_{4n+j} + X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}dx_{4n+j} - X^{5n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}}dx_{4n+j}$$
$$-X^{6n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}}dx_{4n+j} + X^{7n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}}dx_{4n+j} + X^i\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{2n+i}}dx_{5n+j}$$
$$-X^{n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{4n+i}}dx_{5n+j} - X^{2n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_i}dx_{5n+j} + X^{3n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}}dx_{5n+j}$$
$$+ X^{4n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}}dx_{5n+j} - X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}}dx_{5n+j} - X^{6n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}}dx_{5n+j}$$
$$+X^{7n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}dx_{5n+j} + X^i\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}}dx_{6n+j} - X^{n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}}dx_{6n+j}$$
$$-X^{2n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_i}dx_{6n+j} + X^{3n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}}dx_{6n+j} + X^{4n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}}dx_{6n+j}$$
$$-X^{5n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}}dx_{6n+j} - X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}dx_{6n+j} + X^{7n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}dx_{6n+j}$$
$$+X^i\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}dx_{7n+j} - X^{n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}dx_{7n+j} - X^{2n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_i}dx_{7n+j}$$
$$+ X^{3n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}dx_{7n+j} + X^{4n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}dx_{7n+j} - X^{5n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}dx_{7n+j}$$
$$-X^{6n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}dx_{7n+j} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}dx_{7n+j} - \frac{\partial L}{\partial x_j}dx_j - \frac{\partial L}{\partial x_{n+j}}dx_{n+j}$$
$$-\frac{\partial L}{\partial x_{2n+j}}dx_{2n+j} - \frac{\partial L}{\partial x_{3n+j}}dx_{3n+j} - \frac{\partial L}{\partial x_{4n+j}}dx_{4n+j} - \frac{\partial L}{\partial x_{5n+j}}dx_{5n+j} - \frac{\partial L}{\partial x_{6n+j}}dx_{6n+j}$$
$$-\frac{\partial L}{\partial x_{7n+j}}dx_{7n+j}$$

With the use of Eq.(1), the following expressions can be obtained:

$$-X^i \frac{\partial^2 L}{\partial x_j \partial x_{2n+i}} \delta_i^j dx_i + X^i \frac{\partial^2 L}{\partial x_j \partial x_{4n+i}} \delta_i^j dx_{n+i} + X^i \frac{\partial^2 L}{\partial x_j \partial x_i} \delta_i^j dx_{2n+i} - X^i \frac{\partial^2 L}{\partial x_j \partial x_{6n+i}} \delta_i^j dx_{3n+i} -$$
$$X^i \frac{\partial^2 L}{\partial x_j \partial x_{n+i}} \delta_i^j dx_{4n+i} + X^i \frac{\partial^2 L}{\partial x_j \partial x_{7n+i}} \delta_i^j dx_{5n+i} + X^i \frac{\partial^2 L}{\partial x_j \partial x_{3n+i}} \delta_i^j dx_{6n+i} - X^i \frac{\partial^2 L}{\partial x_j \partial x_{5n+i}} \delta_i^j dx_{7n+i}$$
$$-X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{2n+i}} \delta_{n+i}^{n+j} dx_i + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{4n+i}} \delta_{n+i}^{n+j} dx_{n+i} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_i} \delta_{n+i}^{n+j} dx_{2n+i}$$
$$-X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{6n+i}} \delta_{n+i}^{n+j} dx_{3n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{n+i}} \delta_{n+i}^{n+j} dx_{4n+i} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{7n+i}} \delta_{n+i}^{n+j} dx_{5n+i} +$$
$$X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{3n+i}} \delta_{n+i}^{n+j} dx_{6n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{5n+i}} \delta_{n+i}^{n+j} dx_{7n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{2n+i}} \delta_{2n+i}^{2n+j} dx_i +$$
$$X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{4n+i}} \delta_{2n+i}^{2n+j} dx_{n+i} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_i} \delta_{2n+i}^{2n+j} dx_{2n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{6n+i}} \delta_{2n+i}^{2n+j} dx_{3n+i}$$
$$- X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{n+i}} \delta_{2n+i}^{2n+j} dx_{4n+i} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{7n+i}} \delta_{2n+i}^{2n+j} dx_{5n+i} +$$
$$X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{3n+i}} \delta_{2n+i}^{2n+j} dx_{6n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{5n+i}} \delta_{2n+i}^{2n+j} dx_{7n+i}$$
$$- X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{2n+i}} \delta_{3n+i}^{3n+j} dx_i + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{4n+i}} \delta_{3n+i}^{3n+j} dx_{n+i}$$
$$+X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_i} \delta_{3n+i}^{3n+j} dx_{2n+i} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{6n+i}} \delta_{3n+i}^{3n+j} dx_{3n+i} -$$
$$X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{n+i}} \delta_{3n+i}^{3n+j} dx_{4n+i} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{7n+i}} \delta_{3n+i}^{3n+j} dx_{5n+i} +$$
$$X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{3n+i}} \delta_{3n+i}^{3n+j} dx_{6n+i} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{5n+i}} \delta_{3n+i}^{3n+j} dx_{7n+i}$$
$$- X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{2n+i}} \delta_{4n+i}^{4n+j} dx_i + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{4n+i}} \delta_{4n+i}^{4n+j} dx_{n+i} +$$
$$X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_i} \delta_{4n+i}^{4n+j} dx_{2n+i} - X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{6n+i}} \delta_{4n+i}^{4n+j} dx_{3n+i} -$$
$$X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{n+i}} \delta_{4n+i}^{4n+j} dx_{4n+i} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{7n+i}} \delta_{4n+i}^{4n+j} dx_{5n+i} +$$
$$X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{3n+i}} \delta_{4n+i}^{4n+j} dx_{6n+i} - X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{5n+i}} \delta_{4n+i}^{4n+j} dx_{7n+i}$$
$$- X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{2n+i}} \delta_{5n+i}^{5n+j} dx_i + X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{4n+i}} \delta_{5n+i}^{5n+j} dx_{n+i} +$$
$$X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_i} \delta_{5n+i}^{5n+j} dx_{2n+i} - X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{6n+i}} \delta_{5n+i}^{5n+j} dx_{3n+i} -$$
$$X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{n+i}} \delta_{5n+i}^{5n+j} dx_{4n+i} + X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{7n+i}} \delta_{5n+i}^{5n+j} dx_{5n+i} +$$
$$X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{3n+i}} \delta_{5n+i}^{5n+j} dx_{6n+i} - X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{5n+i}} \delta_{5n+i}^{5n+j} dx_{7n+i}$$
$$- X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{2n+i}} \delta_{6n+i}^{6n+j} dx_i + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{4n+i}} \delta_{6n+i}^{6n+j} dx_{n+i} +$$
$$X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_i} \delta_{6n+i}^{6n+j} dx_{2n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{6n+i}} \delta_{6n+i}^{6n+j} dx_{3n+i} -$$
$$X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{n+i}} \delta_{6n+i}^{6n+j} dx_{4n+i} + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{7n+i}} \delta_{6n+i}^{6n+j} dx_{5n+i} +$$
$$X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{3n+i}} \delta_{6n+i}^{6n+j} dx_{6n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{5n+i}} \delta_{6n+i}^{6n+j} dx_{7n+i}$$
$$- X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{2n+i}} \delta_{7n+i}^{7n+j} dx_i + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{4n+i}} \delta_{7n+i}^{7n+j} dx_{n+i} +$$
$$X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_i} \delta_{7n+i}^{7n+j} dx_{2n+i} - X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{6n+i}} \delta_{7n+i}^{7n+j} dx_{3n+i} -$$
$$X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{n+i}} \delta_{7n+i}^{7n+j} dx_{4n+i} + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{7n+i}} \delta_{7n+i}^{7n+j} dx_{5n+i} +$$
$$X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{3n+i}} \delta_{7n+i}^{7n+j} dx_{6n+i} - X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{5n+i}} \delta_{7n+i}^{7n+j} dx_{7n+i} +$$
$$\frac{\partial L}{\partial x_j} dx_j + \frac{\partial L}{\partial x_{n+j}} dx_{n+j} + \frac{\partial L}{\partial x_{2n+j}} dx_{2n+j} + \frac{\partial L}{\partial x_{3n+j}} dx_{3n+j} + \frac{\partial L}{\partial x_{4n+j}} dx_{4n+j} +$$
$$\frac{\partial L}{\partial x_{5n+j}} dx_{5n+j} + \frac{\partial L}{\partial x_{6n+j}} dx_{6n+j} + \frac{\partial L}{\partial x_{7n+j}} dx_{7n+j} = 0$$

If a curve denoted by $\alpha : R \rightarrow R^8$ is considered to be an integral curve of ξ, then we calculate the following equation:

$$-X^i \frac{\partial^2 L}{\partial x_j \partial x_{2n+i}} dx_j - X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{2n+i}} dx_j - X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{2n+i}} dx_j - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{2n+i}} dx_j$$
$$-X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{2n+i}} dx_j - X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{2n+i}} dx_j - X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{2n+i}} dx_j -$$

$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}dx_j + X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{4n+i}}dx_{n+j} + X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{4n+i}}dx_{n+j} + X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{4n+i}}dx_{n+j}$$
$$+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{4n+i}}dx_{n+j} + X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{4n+i}}dx_{n+j} + X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{4n+i}}dx_{n+j} +$$
$$X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}}dx_{n+j} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}dx_{n+j} + X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{i}}dx_{2n+j} +$$
$$X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{i}}dx_{2n+j} + X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{i}}dx_{2n+j} + X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{i}}dx_{2n+j} +$$
$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{i}}dx_{2n+j} + X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{i}}dx_{2n+j} + X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{i}}dx_{2n+j} +$$
$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{i}}dx_{2n+j} + - X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{6n+i}}dx_{3n+j} - X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{6n+i}}dx_{3n+j} -$$
$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{6n+i}}dx_{3n+j} - X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}}dx_{3n+j} - X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}}dx_{3n+j} -$$
$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}}dx_{3n+j} - X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}}dx_{3n+j} - X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}dx_{3n+j} -$$
$$X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{n+i}}dx_{4n+j} - X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{n+i}}dx_{4n+j} - X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{n+i}}dx_{4n+j} -$$
$$X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}}dx_{4n+j} - X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}dx_{4n+j} - X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}}dx_{4n+j} -$$
$$X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}}dx_{4n+j} - X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}dx_{4n+j} + X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{7n+i}}dx_{5n+j} +$$
$$X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{7n+i}}dx_{5n+j} + X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{7n+i}}dx_{5n+j} + X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}}dx_{5n+j} +$$
$$X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}}dx_{5n+j} + X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}}dx_{5n+j} + X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}}dx_{5n+j} +$$
$$X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}dx_{5n+j} + X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{3n+i}}dx_{6n+j} + X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{3n+i}}dx_{6n+j} +$$
$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{3n+i}}dx_{6n+j} + X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}}dx_{6n+j} + X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}}dx_{6n+j} +$$
$$X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}}dx_{6n+j} + X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}dx_{6n+j} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}dx_{6n+j} -$$
$$X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{5n+i}}dx_{7n+j} - X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{5n+i}}dx_{7n+j} - X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{5n+i}}dx_{7n+j} -$$
$$X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}}dx_{7n+j} - X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}}dx_{7n+j} - X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}dx_{7n+j} -$$
$$X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}dx_{7n+j} - X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}dx_{7n+j} + \frac{\partial L}{\partial x_j}dx_j + \frac{\partial L}{\partial x_{n+j}}dx_{n+j} + \frac{\partial L}{\partial x_{2n+j}}dx_{2n+j}$$
$$+\frac{\partial L}{\partial x_{3n+j}}dx_{3n+j} + \frac{\partial L}{\partial x_{4n+j}}dx_{4n+j} + \frac{\partial L}{\partial x_{5n+j}}dx_{5n+j} + \frac{\partial L}{\partial x_{6n+j}}dx_{6n+j} + \frac{\partial L}{\partial x_{7n+j}}dx_{7n+j} = 0 \qquad (13)$$

Or

$$-[X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{2n+i}} + X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{2n+i}} X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{2n+i}} + X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{2n+i}} + X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{2n+i}}$$
$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{2n+i}} + X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}}]\, dx_j + \frac{\partial L}{\partial x_j}dx_j +$$
$$[X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{4n+i}} + X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{4n+i}} + X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{4n+i}} + X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{4n+i}} + X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{4n+i}}$$
$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{4n+i}} + X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}}]\, dx_{n+j} + \frac{\partial L}{\partial x_{n+j}}dx_{n+j} +$$
$$[X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{i}} + X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{i}} + X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{i}} + X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{i}} + X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{i}}$$
$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{i}} + X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{i}} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{i}}]\, dx_{2n+j} + \frac{\partial L}{\partial x_{2n+j}}dx_{2n+j} \; -[X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{6n+i}} + X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{6n+i}} +$$
$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{6n+i}} + X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}} + X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}}$$
$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}} + X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}}]\, dx_{3n+j} + \frac{\partial L}{\partial x_{3n+j}}dx_{3n+j}$$
$$-[X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{n+i}} + X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{n+i}} + X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{n+i}} + X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}} + X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}}$$
$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}} + X^{6n+i}\frac{\partial^2 L}{\partial x_{j}\partial x_{n+i}} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}}]\, dx_{4n+j} + \frac{\partial L}{\partial x_{4n+j}}dx_{4n+j}$$
$$+[X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{7n+i}} + X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{7n+i}} + X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{7n+i}} + X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}} + X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}}$$
$$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}} + X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}} + X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}}]dx_{5n+j} + \frac{\partial L}{\partial x_{5n+j}}dx_{5n+j} +[X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{3n+i}} + X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{3n+i}} +$$
$$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{3n+i}} + X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}} + X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}}$$

$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}}]\,dx_{6n+j}+\frac{\partial L}{\partial x_{6n+j}}dx_{6n+j}-[X^{i}\frac{\partial^2 L}{\partial x_{j}\partial x_{5n+i}}+X^{n+i}\frac{\partial^2 L}{\partial x_{n+j}\partial x_{5n+i}}+$
$X^{2n+i}\frac{\partial^2 L}{\partial x_{2n+j}\partial x_{5n+i}}+X^{3n+i}\frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}}+X^{4n+i}\frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}}$
$+X^{5n+i}\frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}}+X^{6n+i}\frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}}+X^{7n+i}\frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}}]\,dx_{7n+j}+\frac{\partial L}{\partial x_{7n+j}}dx_{7n+j}=0$

In this equation can be concise manner

$$-\sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{2n+i}}dx_j+\frac{\partial L}{\partial x_j}dx_j+\sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{4n+i}}dx_{n+j}+\frac{\partial L}{\partial x_{n+j}}dx_{n+j}$$
$$+\sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{i}}dx_{2n+j}+\frac{\partial L}{\partial x_{2n+j}}dx_{2n+j}-\sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{6n+i}}dx_{3n+j}+$$
$$\frac{\partial L}{\partial x_{3n+j}}dx_{3n+j}-\sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{n+i}}dx_{4n+j}+\frac{\partial L}{\partial x_{4n+j}}dx_{4n+j}+$$
$$\sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{7n+i}}dx_{5n+j}+\frac{\partial L}{\partial x_{5n+j}}dx_{5n+j}+\sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{3n+i}}dx_{6n+j}+$$
$$\frac{\partial L}{\partial x_{6n+j}}dx_{6n+j}-\sum_{a=0}^{7}X^{an+i}\frac{\partial^2 L}{\partial x_{an+j}\partial x_{5n+i}}dx_{7n+j}+\frac{\partial L}{\partial x_{7n+j}}dx_{7n+j}=0 \quad (14)$$

Then we obtain the equations

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right)+\frac{\partial L}{\partial x_{2n+i}}=0\ ,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right)-\frac{\partial L}{\partial x_{4n+i}}=0\ ,\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right)-\frac{\partial L}{\partial x_i}=0\ ,$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right)+\frac{\partial L}{\partial x_{n+i}}=0\ ,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right)+\frac{\partial L}{\partial x_{6n+i}}=0\ ,\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right)-\frac{\partial L}{\partial x_{7n+i}}=0\ ,$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right)-\frac{\partial L}{\partial x_{3n+i}}=0\ ,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right)+\frac{\partial L}{\partial x_{5n+i}}=0 \quad (15)$$

Thus equations obtained in Eq(15) are called Euler-Lagrange equations structured by means of means of $\Phi_L^{J_2}$ on the standard Cliffordian $K\ddot{a}hler$ manifold (R^8,V) and So, the triple$(R^8,\Phi_L^{J_2},\xi)$ is said to be a mechanical system on the standard Cliffordian $K\ddot{a}hler$ manifold (R^8,V).

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