Available online at http://www.journalijdr.com

IJDR

ISSN: 2230-9926

International Journal of Development Research
Vol. 07, Issue, 07, pp.13760-13763, July, 2017

ORIGINAL RESEARCH ARTICLE Open Access

PROPERTIES OF INTUITIONISTIC FUZZY SET OPERATORS

[1,2]Mala, S.K. and [3]Dr. Shanmugapriya, M.M.

[1]PhD Scholar in Mathematics, Karpagam University, Coimbatore, Tamilnadu- 641021
[2]Assistant Professor of Mathematics, KG College of Arts and Science, Saravanampatti, Tamilnadu - 641035
[3]Asst. Professor and Head of Department (i/c) in Mathematics, Karpagam Academy of Higher Education, Coimbatore, Tamilnadu- 641021

ARTICLE INFO

Article History:

Received 05th April, 2017
Received in revised form
24th May, 2017
Accepted 06th June, 2017
Published online 22nd July, 2017

Key Words:

Preliminaries,
Intuitionistic,
Membership.

ABSTRACT

In this paper, various operations in Intuitionistic Fuzzy Sets are discussed. Some theorems are proved for establishing the properties of intuitionistic fuzzy operators with respect to different intuitionistic fuzzy sets.

Corresponding author:

Citation: Mala, S.K. and Dr. Shanmugapriya, M. M. 2017. "Properties of intuitionistic fuzzy set operators", *International Journal of Development Research*, 7, (07), 13760-13763.

INTRODUCTION

Crisp set [5] which has a membership function only 0 and 1 is applied in a lot of branches besides mathematics. To get a wider application of the set theory, L.A. Zadeh [6] introduced the notion of a Fuzzy sub set μ of a Set X as a function from X to [0,1]. After the introduction of Fuzzy sets by L.A.Zadeh [6], the Fuzzy concept has been introduced in almost all branches of Mathematics. Then the concept of Intuitionistic Fuzzy Set (IFS) was introduced by K.T. Atanassov [1] as a generalization of the notation of a Fuzzy set. Here, we discuss the algebraic nature of Intuitionistic Fuzzy operations and prove some results on the commutative Monoid.

1. Preliminaries

For any two IFSs A and B, the following relations and operations can be defined [2, 3, 4] as follows.

Definition 1.1 - Crisp Sets:

The Crisp set is defined in such a way to classify the individuals in the Universe in two groups : Members and Non Members

Definition 1.2 – Fuzzy Sets:

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one.

Definition 1.3 – Fuzzy Sub sets:

Let S be any non empty set, A mapping μ from S to [0,1] is called a Fuzzy sub set of S.

Definition 1.4 – Intuitionistic Fuzzy sets:

Intuitionistic fuzzy sets are sets whose elements have degrees of membership and non-membership. Intuitionistic fuzzy sets have been introduced by Krassimir Atanassov (1983) as an extension of Lotfi Zadeh's notion of fuzzy set, which itself extends the classical notion of a set. An Intuitionistic Fuzzy Set A in a non empty set X is an object having the form

A= {<x, $\mu_A(x), \gamma_A(x)$>/x∈X} where the functions μ_A : X→[0.1] and γ_A : X→[0.1] denote the degrees of membership and non membership of the element x∈X to A respectively and satisfy $0\leq \mu_A(x)+\gamma_A(x)\leq 1$ for all x∈X. The family of all intuitionistic fuzzy sets in X denoted by IFS (X).

Definition: 1.5 – Operators of intuitionistic fuzzy sets

For every two IFSs A and B the following operations and relations can be defined as

A∩B iff (for all x∈E)($\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$
A=B iff (for all x∈E)($\mu_A(x) = \mu_B(x)$)and($\gamma_A(x) = \gamma_B(x)$)
A∩B = {[x,min($\mu_A(x)$, $\mu_B(x)$),max($\gamma_A(x), \gamma_B(x)$)]/x∈E}
A∪B = {[x,max(($\mu_A(x)$, $\mu_B(x)$),min($\gamma_A(x), \gamma_B(x)$))]/x∈E}
A+B= {[x, ($\mu_A(x)+\mu_B(x)-\mu_A(x)\mu_B(x)$, $\gamma_A(x).\gamma_B(x)$)]/ x∈E}
A.B= {[x, ($\mu_A(x)\ \mu_B(x)$, $\gamma_A(x)+\gamma_B(x)-\gamma_A(x).\gamma_B(x)$)]/x∈E}
A@B= { [x, $\mu_A(x)+\mu_B(x)/2$, $\gamma_A(x)+\gamma_B(x)/2$]/x∈E}

2. Proof of theorems

Theorem 2.1

(***A***∩***B***) @ ***C***=(***A*** @ ***C***)∩(***B*** @ ***C***)

PROOF:

We know that,

A∩B = {< x, min { μA(x),μB(x) }, max { γA(x),γ(x) } > / x ϵ E }
A@B={<x, μA(x) + μB (x)/2, γRA(x)+γRB(x)/2>/x∈E}
L.H.S. = (A∩B)@C
= {<x, min{ μRA(x),μRB(x)}, max{γRA(x),γRB(x)}/x∈E} @
{<x,μC(x),γC(x)>/xϵE}
Let μA(x) < μB(x) and γB(x) >γA(x) --------------> *
= {< x, μA(x), γB(x) > / x ϵ E } @ { < x, μC(x), γC(x) > / x ϵ E }
= {< x, μA(x) + μC(x)/2 , γB(x) + γC(x)/2 > / x ϵ E} ------------> ①
R.H.S = (A @ C) ∩(***B***@***C***)
= {< x, μA(x) + μC(x)/2 , γA(x) + γC(x)/2 > / x ϵ E } ∩
{< x, μB(x) + μC(x)/2 , γB(x) + γB(x)/2 > /x ϵ E}
= {< x, min {μA(x) + μB(x) + μC(X)/2} ,
max {(γA(x) + γC(x))/2 ;(γB(x) + γC(x))/2 } / x ϵ E }
= {< x, μA(x) + μC(x)/2 , γB(x) + μC(x)/2> / x ϵ E } by * --------> ②
From ① and ② L.H.S = R.H.S
Hence (*A*∩*B*)@*C*=(*A* @ *C*) ∩(*B* @ *C*)
Similarly we prove that
A @ (*B* ∩*C*)=(*A* @ *B*) ∩(*A* @ *C*)

Theorem 2.2

(***A*** ∪***B***)@ ***C***=(***A*** @ ***C***) ∪(***B*** @ ***C***)
L.H.S. = (***A*** ∪***B***) @ ***C***
= {< x, max { μA(x), μB(x) }, min { γA(x), γB(x) / x ϵ E } @
{< x, μC(x), γC(x) > / x ϵ E }
Let μA < μB , γA(x) < γB(x) ---------------> (**)
= {< x, μB(x), γA(x) > / x ϵ E} @ {< x, μC(x), γC(x) > / x ϵ E }

= {< x, μB(x) + μC(x)/2 , γA(x) + γC(x)/2 > / x ϵ E } -----------> ③
R.H.S. = (**A** @) ∪(**B** @ **C**)
= {< x, μA(x) + μC(x)/2 , γA(x) + γC(x)/2 > / x ϵ E } ∪
{< x, μB(x) + μC(x)/2 , γB(x) + γC(x)/2 > / x ϵ E }
= {< x, max { μA(x) + μC(x)/2 , μB(x) + μC(x)/2} ,
min { γA(x) + γC(x)/2 , γB(x) + γC(x)/2 } / x ϵ E } by (**)
= {< x, μB(x) + μC(x)/2 , γA(x) + γC(x)/2 > / x ϵ E} ------------> ④
③ =④
⟹ L.H.S. = R.H.S
Similarly we prove that
A @ (**B** ∪**C**)=(**A** @ **B**) ∪**A** @ **C**

Theorem 2.3

(**A** @ **B**) .**C**=**A**.**C** @ **B**.**C**
L.H.S. = (A @ B) . C
= {< x , μ_A (x) + μ_B(x)/2 , γ_A(x) + γ_B(x)/2 > / x ϵ E } .
{< x, μ_C(x) , γ_C(x) >/ x ϵ E}
= {< x, μ_C(x) . [μ_A(x) + μ_B(x)]/2 , [γ_A(x) + γ_B(x)]/2 + γ_C –
[γ_A(x) + γ_B(x)] /2 . γ_C(x) / x ϵ E} -------------> ⑤
R.H.S. = A . C @ B . C
= {< x, μ_A(x) . μ_C(x) , γ_A(x) + γ_C(x) – γ_A(x) . γ_C(x) > / x ϵ E } @
{< x, μ_B(x) . μ_C(x), γ_B(x) + γ_C(x) – γ_B(x) . γ_C(x) >/ x ϵ E}
= {< x, [μ_A(x) . μ_C(x) + μ_B(x) . μ_C(x)]/2 , γ_A(x) + γ_C(x) –
γ_A(x) . γ_C(x) + γ_B(x) + γ_C(x) – γ_B(x).γ_C(x)/2 > / x ϵ E }
= {< x, [μ_A(x) + μ_B(x)]/2 . μ_C(x) , {γ_A(x) + γ_B(x) + 2γ_C(x) –
γ_C(x) [γ_A(x) + γ_B(x)] }/2 / x ϵ E }
= {< x, [μ_A(x) + μ_B(x)]/2} . μ_C(x) , [γ_A(x) + γ_B(x)]/2 + γ_C(x) –
γ_C(x) [γ_A(x) + γ_B(x)]/2 / x ϵ E} ------------> ⑥
⑤ = ⑥
Hence proved.

Theorem 2.4

(∩*B*) .*C* = {< x, min (μ_A(x) , μ_B(x)) , max (γ_A(x) , γB(x) >/ x ϵ E } .
{< x , μ_C(x) ,γ_C(x) >/ x ϵ E }
Let μ_A(x) < μ_B(x) and γ_A(x) > γ_B(x) ---------> ①
Then,
(A ∩ B) . C = {< x, μ_A(x), γ_A(x) > / x ϵ E } . {< x, μ_C(x), γ_C(x) > / x ϵ E }
= {< x, μ_A(x).μ_C(x), γ_A(x)+γ_C(x)–γ_A(x)γC(x)>/ x ϵ E } --->②
(A . C) ∩ (B. C)
= {< x, μ_A(x), γ_A(x) >/ x ϵ E} . {< x, μ_C(x), γ_C(x) >/ x ϵ E} ∩
{< x, μ_A(x), γ_B(x) > / x ϵ E } . {< x, μ_C(x), γ_C(x) >/ x ϵ E }
= {< x, μ_A(x)μ_C(x) , γ_A(x)+γ_C(x)–γ_A(x)γ_C(x) > / x ϵ E } ∩
{< x, μ_B(x)μ_C(x) , γ_B(x)+γ_C(x)–γ_B(x)γ_C(x) > / x ϵ E }
= {< x, min { μ_A(x)μ_C(x) , μ_B(x)μ_C(x) } max { γ_A(x)γ_C(x)–
γ_A(x)γ_C(x) , γ_B(x)+γ_C(x)–γ_B(x)γ_C(x) } > / x ϵ E }
Since μ_A(x) < μ_B(x) , γ_A(x) > γ_B(x) by ①
= {< x, μ_A(x)μ_C(x) , γ_A(x)+γ_C(x)–γ_A(x)γ_C(x) / x ϵ E } -----> ②
① = ②
(*A* ∩*B*) .*C*=(*A* .*C*)∩(*B* .*C*)
Hence proved.

Theorem 2.5

(**A** ∩**B**)+(**A** ∪**B**)=**A**+**B**
L.H.S. = (∩**B**)+(**A** ∪**B**)
= {< x, min { μA(x),μB (x) }, max { γA(x),γB(x) }/ x ϵ E } +
{< x, max { μA(x),μB(x) }, min { γA(x),γB(x) } / x ϵ E}
Let μA(x) < μB(x) and γA(x) < γB(x) --------------------> *
= {< x, μA(x),γB(x) > / x ϵ E } + {< x, μB(x),γA(x) > / x ϵ E }
= {< x, μA(x)+μB(x)–μA(x)μB(x),γA(x)γB(x) > / x ϵ E }
= A + B by definition of " + "
= R.H.S – Hence proved.

Theorem 2.6

($\boldsymbol{A} \cap \boldsymbol{B}$) .($\boldsymbol{A} \cup \boldsymbol{B}$)= $\boldsymbol{A}$.$\boldsymbol{B}$
L.H.S. = ($\cap \boldsymbol{B}$) .($\boldsymbol{A} \cup \boldsymbol{B}$)
= {< x, min { $\mu_A(x),\mu_B(x)$ }, max { $\gamma_A(x),\gamma_B(x)$ } / x ϵ E } .
{< x, max { $\mu_A(x),\mu_B(x)$ }, min { $\gamma_A(x),\gamma_B(x)$ } / x ϵ E }
= {< x, $\mu_A(x),\gamma_B(x)$ > / x ϵ E } . {< x, $\mu_B(x),\gamma_A(x)$ > / x ϵ E } by *
= {< x, $\mu_A(x)\mu_B(x)$, $\gamma_B(x)+\gamma_A(x) - \gamma_B(x)\gamma_A(x)$ > / x ϵ E }
= {< x, $\mu_A(x)\mu_B(x)$, $\gamma_A(x)+\gamma_B(x) - \gamma_A(x)\gamma_B(x)$ > / x ϵ E }
= A . B by definition of " . "
= R.H.S – Hence proved.

Theorem 2.7

(A+B) @ (A .B)=A @ B
L.H.S. = (+ $\boldsymbol{B}$) @ ($\boldsymbol{A}$.$\boldsymbol{B}$)
= {< x, $\mu_A(x)+\mu_B(x) - \mu_A(x).\mu_B(x)$, $\gamma_A(x)\gamma_B(x)$ > / x ϵ E } @
{< x, $\mu_A(x).\mu_B(x)$, $\gamma_A(x)+\gamma_B(x)-\gamma_A(x)\gamma_B(x)$ > / x ϵ E }
= {< x, { $\mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x)+\mu_A(x).\mu_B(x)$ } / 2 ,
{$\gamma_A(x).\gamma_B(x)+\gamma_A(x)+\gamma_B(x)-\gamma_A(x)\gamma_B(x)$ } / 2 > / x ϵ E }
= {< x, $\mu_A(x)+\mu_B(x)$ / 2 , $\gamma_A(x)+\gamma_B(x)$ / 2 > / x ϵ E }
= A @ B by definition.
= R.H.S – Hence proved.

Theorem 2.8

($\boldsymbol{A} \cap \boldsymbol{B}$) @ ($\boldsymbol{A} \cup \boldsymbol{B}$)=$\boldsymbol{A}$ @ $\boldsymbol{B}$
L.H.S.= ($\boldsymbol{A} \cap \boldsymbol{B}$) @ ($\boldsymbol{A} \cup \boldsymbol{B}$)
= {< x, min { $\mu_A(x),\mu_B(x)$ } , max { $\gamma_A(x),\gamma_B(x)$ } > / x ϵ E } @
{< x, max { $\mu_A(x),\mu_B(x)$ } , min { $\gamma_A(x),\gamma_B(x)$ } > / x ϵ E }
Let $\mu_A(x) < \mu_B(x)$ and $\gamma_A(x) < \gamma_B(x)$
= {< x, $\mu_A(x),\gamma_B(x)$ > / x ϵ E } @ {< x, $\mu_B(x),\gamma_A(x)$ > / x ϵ E }
= {< x, $[\mu_A(x)+\mu_B(x)]/2$, $[\gamma_B(x)+\gamma_A(x)]/2$ > / x ϵ E }
= A @ B
=R.H.S – Hence proved.

Theorem 2.9

($\boldsymbol{A} \cap \boldsymbol{B}$) @ ($\boldsymbol{A} \cup \boldsymbol{B}$)=($\boldsymbol{A}$+$\boldsymbol{B}$) @ ($\boldsymbol{A}$.$\boldsymbol{B}$)
From theorem 7 and 8
L.H.S. = R.H.S.

Conclusion

We have defined different operations of Intuitionistic Fuzzy Sets. Using these, we have proved different relations between these operators in the intuitionistic fuzzy sets.

REFERENCES

Atanassov, K. T, Intuitionistic Fussy Sets, Fussy Sets and Systems, (1986)., 1973
Atanassov, K. T, Intuitionistic Fussy Sets, VII ITKR's Session, Sofia (June 1983)
Atanassov, K. T, Intuitionistic Fussy Sets, Fussy Sets and Systems, (1986)., 1973
Atanassov, K. T, New Operations Defined Over the Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems (1994)
George J.Klir &Tina A.Folger , Fuzzy Sets ,Uncertainty & Information .
Zadeh. L. A, Fuzzy Sets., Information Control. Vol. 8 (1965).

****