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NUMBER OF WINDOWS IN A THREE- CONNECTED CUBIC BIPARTITE PLANAR GRAPH

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ABSTRACT

A planar graph is one that can be drawn on a two-dimensional plane such that no two edges cross. A cubic graph is one in which all vertices have degree three. A three connected graph is one that cannot be disconnected by removal of two vertices. A graph is bipartite whose vertices can be colored using exactly two colors such that no two adjacent vertices have the same color. In this paper I shall prove that the number of windows in every cubic three connected planar bipartite graph of n nodes is $\frac{n+4}{2}$. where n is the number of nodes.

INTRODUCTION

A planar graph is one that can be drawn on a two-dimensional plane such that no two edges cross. A cubic graph is one in which all vertices have degree three. A three connected graph is one that cannot be disconnected by removal of two vertices. A graph is bipartite whose vertices can be colored using exactly two colors such that no two adjacent vertices have the same color. [1] pp 16-20 A plane representation of a graph divides the plane into regions also called windows, faces or meshes. A window is characterized by the set of edges or the set of nodes forming its boundary. Note that windows is not defined in a non planer graph or even in a planer graph not embedded in a plane. Thus a window is a property of the specific plane representation of a graph. The window of a graph may be finite or infinite. The portion of a plane lying outside a graph embedded in a plane is infinite in it's extend. Since a planar graph may have different plane representation. Euler gives formula for number of windows in a planar graph. [2] pp 88-100

Beauty of the result

Euler gives formula for finding the number of windows of a planar graphs as (e-v+2=w) which is dependent on both nodes and edges thus we can find the number of windows of a planar graph only if we know both nodes and edges, because in all planer graphs nodes and edges are not properly connected by certain relations but in our case when the graph is cubic bipartite and polyhedral both nodes and edges are interrelated one another, we can find the number of nodes of a graph if we know number of edges. Similarly we can find the number of edges of a graph if we know number of nodes, which is not possible in all planar graphs. Thus in nutshell we can find the number of windows in such graphs if we know only nodes (edges) of a graph.

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Lemma 1.1 [1]

A graph can be embedded in a surface of a sphere if and only if it can be embedded in a plane.

Lemma 1.2 [1]

A planer graph may be embedded in a plane such that any specified region can be made the infinite region.

lemma1.3 [1] [2] (Euler theorem)

A connected planer graph with n vertices and e edges has e –n +2 regions.

lemma1.4 [3]

A plane graph is bipartite if and only if each of its faces has an even number of sides.

Lemma 1.5 [1]

A graph is bipartite if and only if it is without odd cycles.

Statement of the result

The number of windows in every cubic three connected planer bipartite graph of n nodes is $\frac{n+4}{2}$.

Proof

Since the graph is cubic planar three connected and bipartite, we know that any cubic bipartite polyhedral graph the number of nodes is even because in bipartite graphs odd cycles are not allowed. Degree of each node is exactly equal to three as graph is cubic. Thus the sum of all the degree of the Graph is 3n that is $\sum_{i=1}^{n} d_i = 3n$ since each edge contributes two to the degrees thus the number of edges in the graph is

E= $\sum_{i=1}^{n} di$ /2

E = $\frac{3n}{2}$ where n is even being bipartite graph.

Thus we conclude that if number of nodes is n number of edges is $\frac{3n}{2}$ and if number of edges is $\frac{3n}{2}$ the number of nodes is $\frac{2}{3}e = \frac{2}{3}\frac{3n}{2} = n$ Now from lemma 1.3 above the number of windows in a connected planer graph is given by

W = e - v + 2 i

Since we have a graph of n nodes as we know it is cubic bipartite and polyhedral the number of edges in such a graph is $\frac{3n}{2}$ as shown above now substitute these values in equation i we get.

W = e - n + 2

W = $\frac{3n}{2}$ - n + 2

W = $\frac{3n-2n}{2}$ + 2

W = $\frac{n}{2}+2$

W = $\frac{n+4}{2}$ that proves the result.

Thus from the above result we conclude that in every cubic bipartite polyhedral graph it is true that

e - v + 2 = $\frac{n+4}{2}$

The above result is not true for other planer graphs as we can take a counter example of three connected bipartite planar graph known as Herschel graph which contain 11 nodes and 18 edges. Contain 9 windows. Does not satisfy the above result.

Future work

Presently I am working on Barnett's conjecture on cubic planar bipartite polyhedral graphs and hope to settle it in alternative form because it is very tough problem cannot be solved by direct method so alternative way is the best possible to settle it.

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